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**STRESS STRAIN BEHAVIOUR OF
ANISOTROPICALLY CONSOLIDATED GANGA SAND**

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By
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to the
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CERTIFICATE

Certified that the work on 'Stress-Strain Behaviour of Anisotropically Consolidated Ganga Sand' by Prabhash Singh has been carried out under my supervision and that his work has not been submitted elsewhere for the award of any degree or diploma.

April, 1981


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NOTATION

| | |
|----------------------|--|
| A | - stress path |
| B | - stress path, model parameter |
| B ₁ | - stress path |
| C | - stress path, model parameter |
| C ₁ | - stress path |
| D ₁ | - model parameter |
| D ₃ | - model parameter |
| E | - stress path |
| E ₁ | - stress path |
| e | - void ratio |
| e _{min} | - minimum void ratio |
| e _{max} | - maximum void ratio |
| G | - specific gravity |
| K ₀ | - coefficient of earth pressure at rest |
| M | - critical state frictional parameter |
| p' | - mean effective stress $(\sigma'_1 + 2 \sigma'_3)/3$ |
| p' _f | - mean effective stress at failure |
| p' _i | - initial value of mean effective stress prior to shearing |
| q | - deviatoric stress $(\sigma'_1 - \sigma'_3)$ |
| q _f | - deviatoric stress at failure |
| u | - pore water pressure |
| α_1, α_3 | - model parameters |
| β_1, β_3 | - model parameters |

| | |
|---------------------------------------|--|
| ϵ_1 | - axial strain |
| ϵ | - shear strain $\frac{2}{3}(\epsilon_1 - \epsilon_3)$ |
| ϵ_v | - volumetric strain $(\epsilon_1 + 2 \epsilon_3)$ |
| η | - stress ratio (q/P') |
| κ | - slope of swelling line in $e - \ln P'$ plot |
| λ | - slope of isotropic compression line in $e - \ln P'$ plot |
| σ'_v | - vertical effective stress |
| σ'_1 | - effective major principal stress |
| σ'_3 | - effective minor principal stress |
| $dP', \delta P'$ | - change in P' |
| $dq, \delta q$ | - change in q |
| $d \epsilon_v, \delta \epsilon_v$ | - increment in volumetric strain |
| $d \epsilon_v^p, \delta \epsilon_v^p$ | - increment in plastic volumetric strain |
| $d \epsilon_v^e, \delta \epsilon_v^e$ | - increment in recoverable volumetric strain |
| $d\epsilon, \delta\epsilon$ | - increment in shear strain |
| $d \epsilon_1, \delta \epsilon_1$ | - increment in axial strain |
| $d\eta, \delta\eta$ | - change in η |
| $dw, \delta w$ | - dissipated energy per unit volume |

SYNOPSIS

Most of the researchers have studied the deformation behaviour of isotropically consolidated granular soils, however, in the field soils seldom experience isotropic stress history. Keeping this in view the deformation behaviour of anisotropically consolidated micaceous Ganga sand has been investigated to check the applicability of (a) Modified Cam-Clay model (Burland 1965) and (b) Mathur (1975) and Jain's (1979) model for prediction of volumetric and shear strain. Drained stress controlled stress paths tests were performed on loose sand samples. The experimental results indicate that the prediction by modified Cam-Clay model do not agree with the experimental results. The semi-empirical model proposed by Mathur (1975) and Jain (1979), predicts the stress-strain behaviour of sand fairly accurately for a wide variety of stress paths. The stress-strain behaviour of sand investigated generally confirms the findings of Tatsuoka and Ishihara (1972) in respect of: (a) shear strain contours being independent of stress path in $\tau - P'$ space (b) strain increment ratio $(\delta\epsilon_v/\delta\epsilon)$ being stress path dependent and is unique only when the critical state approaches. Oda's (1972) findings regarding the linear relationship between stress ratio σ_1/σ_3 and strain increment ratio $(\delta\epsilon_v/\delta\epsilon_1)$, is confirmed in this investigation, however, it is observed from the experimental results that this relationship is stress path dependent.

CHAPTER 1

INTRODUCTION

In many engineering problems it is necessary to know with accuracy both the strength and the deformation behaviour of material in use. Satisfactory theories and consequent prediction of behaviour are dependant on making simplifying assumptions and approximations about the observed stress-strain behaviour.

The simplest and most widely used assumption is that a particular material is perfectly elastic so that it has a linear stress-strain characteristic as shown in Figure 1(a). This assumption is valid to a high degree of accuracy for many engineering materials under their condition of normal use so that the classical theory of elasticity has not unnaturally dominated the subject and tended to discourage development of other complimentary theories.

For the case of ductile materials such as mild steel capable of undergoing large plastic deformation beyond the elastic limit, this elastic assumption has either to be supplemented by assuming elastic-plastic behaviour as shown in Figure 1(b) or replaced by assuming rigid-plastic behaviour as typified in Figure 1(c).

In soil mechanics the observed stress strain behaviour of soil is generally of the form shown in Figure 1(d). In curve (1) there is a distinct peak or failure

stress which is typical of dense sand or overconsolidated clays whereas the curve (2) does not have a peak and is typical of loose sand or normally consolidated clay. Neither curve approximates at all closely the ideal behaviour of Figure 1(a) to (c).

Until recently (1960) nearly all soil mechanics investigations have ignored general stress-strain behaviour and concentrated almost exclusively on the peak or ultimate strength. Many designs which are essentially deformation problem have been treated from strength aspect by some simple balance between shear stress and factored strength. While this method is generally successful, soil mechanics require a greater understanding of stress-strain relation. The importance of strains in soil mechanics has been emphasized by Roscoe (1970).

For want of better approximations the earlier theories of earth pressure developed from original work of Coulomb (1776), Rankine (1857) and many others require soil to be perfectly elastic solids while recent work notably by Sokolovsky (1960) and De Josselin de Jong (1959) have made use of theory of perfectly plastic solids. However, neither approach to the problem is likely to be very accurate.

Many researchers have tried to put forward various mathematical, empirical and semiempirical models to describe the stress-strain behaviour of soil. The work due to

late Prof. Roscoe and others was principally a theory of deformation of clays and they have used the same for sands as well. The work of Rowe 'The stress-dilatancy relation for static equilibrium of an assembly of particles in contact' was mainly concerned with deformation of granular materials.

In complex soil conditions where the determination of relevant deformation and strength parameters may involve several inaccuracies, the use of crude stress-strain relationships i.e. incrementally Hookean Duncan Chang model and elastic-perfectly plastic Mohr-Coulomb model is justified. On the other hand in regular soil strata created by sedimentation, a more accurate analysis is possible. It is for these soils that many mathematical, empirical and semi-empirical models have been developed both for clays and sands.

CHAPTER 2

LITERATURE REVIEW

2.1 General

In recent years many investigators have considered deformation behaviour of soil, the main line of research being the formulation of model for soils considering it as an elasto-plastic material with isotropic hardening. It seems that the development of soil models considering isotropic hardening was started at Cambridge University. The Granta-Gravel and the Cam-Clay models became wellknown through the book by Schofield and Wroth (1968) and through the papers, Roscoe and Burland (1968), Calladine (1971) etc. In recent research papers and conferences several models can be found which are conceptually similar to the Cam-Clay model. They are referred to as Cambridge stress-strain models or Critical State models. In these models the plasticity concept is narrowed by some additional assumptions.

One of the assumptions of Cam-Clay model is that the flow rule follows the normality condition. Another assumption is that the position of yield surface in stress space is determined by the density of the soil i.e. volumetric strain serves as a hardening parameter. Furthermore, recoverable shear strain is zero. It has been shown by various researchers on the basis of experimental evidence

that the first two assumptions do not apply atleast for sands. In case of sands it has been well established by Oda (1972), Matsuoka (1974) and Vermeer (1980) that stress dilatancy theory originally proposed by Rowe provides a satisfactory flow rule. Nevertheless, the Cam-Clay concepts are still used by Atkinson and Bransby (1978) and Nova and Wood (1979) although the latter suggest a non-associated flow rule.

Some other models for sand not based on Cam-Clay concepts have been developed. They may be named as Mohr-Coulomb type models since the successive yield surfaces are determined by the mobilised angle of internal friction. Pooroshasb et al (1967), Cole (1967) studied the behaviour of sand on yielding. Vermeer (1980) proposed a double hardening model which is basically an elasto-plastic model for initial loading, unloading and reloading of sand incorporating non-associated flow rule and Rowe's stress dilatancy equation. Furthermore, Vermeer (1980) has quoted the results of the experiments conducted by Gudehus (1980) on isotropically consolidated samples of sands from which it is shown that elasto-plastic model yields good results for loading paths but such models are inadequate for unloading paths. Duncan Chang (1970) developed a model which combine non linear shear stress formulation with a Hookean law for increment of stress and strain. It is obvious that the incrementally Hookean approach does not account for contraction and dilation (the dilatancy behaviour) in deviatoric stress path.

Another important objection to the Duncan-Chang model concerns the direction of the strain rate during loading. According to experiments strains rate are more or less in the direction of existing stresses as expressed by the idealisation that $\dot{\epsilon}_{ij}^p$ is coaxial to σ_{ij} as in plastic model and not to $\dot{\sigma}_{ij}$ as implied by the incrementally Hookean approach. Lade (1977) developed an elasto-plastic model with a shear yield surface. However, the number of experimental constitutive constants required in this model are 14 thus making the use of this model for practical problems quite inconvenient. Besides this model as shown by Vermeer (1980) strictly applies for **drained** tests on isotropically consolidated samples only.

Tatsuoka (1972) performed systematic studies on sands both for loose and dense state in triaxial apparatus. He showed that the relationship between stress ratio q/P' and strain increment vector $\delta\epsilon_v^p / \delta\epsilon^p$ is unique only when the peak approaches otherwise it is path dependent. Further, he has established that the shear strain is unique for a combination of stress ratio q/P' and P' independent of stress path.

Most of the experiments reported by various researchers as mentioned above are on sand samples having isotropic stress history (isotropically consolidated samples). But the natural deposits seldom experience the isotropic stress history. Furthermore, the importance of anisotropic

consolidation prior to shear testing has been well brought out by Campanella and Vaid (1972), Berre and Bjerrum (1973), Yudhbir et al (1975) and Mathur and Yudhbir (1978).

2.2 Deformation Behaviour Based on the Theory of Elasticity and Plasticity

2.2.1 Theory of Elasticity Methods

In the past it was generally assumed that soil is homogeneous isotropic and linearly elastic and theory of linear elasticity was employed for analysis of soil behaviour. Theory of elasticity assumes complete decoupling of volume changes and changes in shear stress. The volumetric strains are caused only by spherical stress and shear strains are caused only by deviatoric stress. Based on the theory of elasticity Duncan and Chang (1970) using Kondner's (1963) findings that the plot of stress and strain in triaxial compression test is very nearly a hyperbola, developed a non linear elastic stress and strain relationship and implemented the same in finite element analysis of embankments and excavations. However, it has been observed experimentally that:

- (1) Most soil deform inelastically almost immediately on application of stress.
- (2) There is coupling between volume changes and changes in shear stress.
- (3) The straining of soil element approaching failure is clearly anisotropic.

For these reasons models based on the theory of elasticity and extension thereof cannot be expected to account properly for soil behaviour specially dilatancy.

2.2.2 Theory of Plasticity Methods

It is wellknown that this theory is based on the mathematical theory of plasticity taking the special characteristics of soil into account. This theory is due to late Prof. Roscoe and others of Cambridge group and has been developed for clay. However, Roscoe and others suggested that this theory would be equally applicable to sand if some modifications are added. The basic tenets of this theory are:

- (a) Yield function: When the state of stress of material is represented by a point in the stress space, the border of elastic and plastic regions is represented by a surface in this space called yield surface and equation of this surface is designated as yield function. As long as the stress point remains in the inner side of this surface enclosed by the region, the material is elastic.
- (b) Hardening rule: Almost all kinds of material show hardening property when deformed. For representing the hardening law some parameter which varies during loading process, is inserted in the yield function thereby changing the location of yield surface as the stress point moves along the stress path.

- (c) Flow rule: The relationship between the increment of plastic strain components and the stress components is represented by the flow rule. If the ratio of each plastic strain component depends only on the stress component and not on the increment of stress component, that is the increment of plastic strain is independent of stress path then the yield function is said to be the plastic potential.

A brief review of various models based on this approach, is given below.

2.2.2.1 Cam-Clay Model

In this model the dissipated energy δw per unit volume of material when undergoing deformation $(\delta \epsilon_v, \delta \epsilon)$ at any state (P', q, e) corresponding to a point on the state boundary surface is $MP' \delta \epsilon$

$$\text{that is } P' \delta \epsilon_v^P + q \delta \epsilon^P = \delta w$$

$$\text{and } \delta w = MP' \delta \epsilon^P$$

Another assumption is that the plastic strain increment vector is normal to the yield surface at the stress point under consideration. This model also assumes that recoverable shear strain is zero. The recoverable volumetric strain is associated with changes in P' through the idealised swelling formula

$$\delta \epsilon_v^G = \frac{\kappa}{1+e} \frac{\delta P'}{P'}$$

where κ is the slope of swelling line in e vs. $\ln P'$ plot.

Making use of above condition and flow rule the strain increment ratio is

$$\frac{\partial \epsilon_v^p}{\partial \epsilon} = M - \eta \quad ,$$

Incremental volumetric strain

$$\delta \epsilon_v = \frac{1}{1+e} \left[\frac{\lambda - \kappa}{M} \delta \eta + \frac{\lambda \delta P'}{P'} \right] \quad \dots 2.1$$

and incremental shear strain

$$\delta \epsilon = \frac{\lambda - \kappa}{1+e} \left[\frac{P' \delta \eta + M \delta P'}{M P' (M - \eta)} \right] \quad \dots 2.2$$

The prediction of volumetric and shear strain were checked by various researchers and found that Cam-Clay model overpredicted strain increment at smaller values of η even-though excellent agreement was found for changes in η at larger values of η .

2.2.2.2 Modified Cam-Clay Model

Burland (1965) modified the expression for work dissipation per unit volume by taking into consideration that even under isotropic stress condition there is dissipation of work and therefore

$$\delta w = P' \sqrt{(\delta \epsilon_v^p)^2 + (M \delta \epsilon^p)^2}$$

Combining this equation with $\delta w = P' \delta \epsilon_v^p + q \delta \epsilon^p$

the strain increment ratio is

$$\frac{\delta \epsilon_v^p}{\delta \epsilon^p} = \frac{M^2 - \eta^2}{2\eta}$$

All other assumption of Cam-Clay were retained. Using the modified energy equation as proposed by Burland (1965) the incremental volumetric and shear strain are given as

$$\delta \epsilon_v = \frac{1}{1+e} \left[\frac{(\lambda - \kappa) 2\eta \partial \eta}{M^2 + \eta^2} + \frac{\lambda \delta P'}{P'} \right] \quad \dots 2.3$$

$$\delta \epsilon = \frac{\lambda - \kappa}{1+e} \left[\frac{2\eta \partial \eta}{M^2 + \eta^2} + \frac{\delta P'}{P'} \right] \left(\frac{2\eta}{M^2 - \eta^2} \right) \quad \dots 2.4$$

Mathur (1975) showed that for K_0 -consolidated samples this model could not predict strains accurately for E_1 , A and B stress-paths (Figure 3) and further demonstrated that this model overpredicts shear strains.

2.2.2.3 Roscoe and Burland (1968) Model

Roscoe and Burland (1968) showed experimentally that elastic limit line lying on the state boundary surface is good approximation for volumetric strains while considerable plastic shear strain does take place for state paths beneath the state boundary surface. Accordingly the modified equation for incremental shear strain is

$$\partial \epsilon^p = \frac{\lambda - \kappa}{1 + e} \left[\frac{2\eta}{M^2 - \eta^2} \right] \left\{ \frac{2\eta \partial \eta}{M^2 + \eta^2} + \frac{\delta P'}{P'} \right\} + \delta(\epsilon^p)_{\epsilon_v^p}$$

The last term was obtained experimentally. For isotropically consolidated samples this modification gave good prediction in the beginning thereafter modified Cam-Clay showed better prediction. But it has been shown by Newland (1973) that this model gives larger prediction of shear strains in comparison to Burland's (1965) model for q -constant and undrained test on anisotropically consolidated samples.

In the above models the basic parameters λ , κ and M have been assumed to be soil constants. Newland (1973), Yudhbir, Mathur and Kuganathan (1978) have shown that parameters $D = (1 - \frac{\kappa}{\lambda})/M$ and λ are not really soil constants and D value differs for drained and undrained condition and λ depends on η .

It is obvious that theory of plasticity can handle the soil behaviour more realistically as compared to the theory of elasticity. However, all these models use associated flow rule though experimentally it is found to be not true. Various researchers have shown that plastic strain increment vectors are dependent on stress paths. Lade and Duncan (1975) suggested use of non-associated flow rule for cohesionless soils.

2.2.3 Semi Empirical Models

Roscoe and Pooroshasb (1963), Wroth (1968) and Newland (1973) developed semi empirical models to overcome the deficiencies of mathematical models mentioned above and tried to obtain the best possible fit to the observed stress-strain behaviour of soil. It has been shown by Yudhbir et al (1975) and Yudhbir and Mathur (1978) that none of these models predicted anisotropically consolidated behaviour close to the actual behaviour. Wroth's (1968) model appears to be more general in that it is able to handle loading and unloading stress paths but only for isotropically consolidated samples.

2.2.3.1 Mathur (1975) and Jain (1979) Model

Observing the fact that soils in general exhibit the coupled behaviour, Mathur (1975) developed a semi empirical approach to predict the stress-strain behaviour for K_0 -Consolidated soil samples. He proposed that a stress path can be simulated by small probes in the direction of P' -constant (pure shear) and q -constant (consolidation) stress paths starting from K_0 -line.

The changes in the volumetric and shear stresses as a result of the application of probe ($\delta P'$, δq) under drained condition, may be expressed as

$$\delta \epsilon_v = \frac{\partial \epsilon_v}{\partial P'} \cdot \delta P' + \frac{\partial \epsilon_v}{\partial q} \cdot \delta q \quad \dots 2.5$$

$$\text{and} \quad \delta \epsilon = \frac{\partial \epsilon}{\partial P'} \cdot \delta P' + \frac{\partial \epsilon}{\partial q} \cdot \delta q \quad \dots 2.6$$

Here $\delta \epsilon_v$ and $\delta \epsilon$ are incremental volumetric and shear strains respectively due to probe ($\delta P'$, δq). The parameters $\frac{\partial \epsilon_v}{\partial P'}$ and $\frac{\partial \epsilon}{\partial P'}$ are to be determined from q -constant test while parameters $\frac{\partial \epsilon_v}{\partial q}$ and $\frac{\partial \epsilon}{\partial q}$ are to be determined from P' -constant test. Here it is assumed that the total effect of any loading increment comprises of the separate effects of the corresponding increase in P' and q . Mathur (1975) showed that this model predicts drained behaviour of anisotropically consolidated samples along all the stress paths quite satisfactorily. However, Mathur checked this model for smaller probes only. Jain (1979) extended this model for predicting the behaviour of lightly overconsolidated clays and streamlined the method of determination of parameters and used the model for prediction of stress-strain behaviour of K_0 -consolidated clays for wide variety of stress paths and found close similarity between predicted and actual behaviour.

2.3 Scope of Investigation

From the preceding review of the literature it becomes clear that most of the constitutive models for prediction of stress strain behaviour of sand are either based on Cam Clay with certain modification or semi empirical models which assume isotropic consolidation of soil. In

field condition the soil is anisotropically consolidated and it has been shown by Yudhbir and Mathur (1978) that both the critical state models and some other semi-empirical models do not satisfactorily predict the stress-strain behaviour of anisotropically consolidated clay samples. In most of the cases the shear strain is very much overpredicted. Mathur (1975) and Jain (1979) model predicts anisotropically consolidated stress-strain behaviour of normally and lightly overconsolidated clay samples for a wide variety of stress paths reasonably well. In case of sands hardly any data is available in literature on the stress-strain behaviour of anisotropically consolidated samples. The currently proposed predictive models may be listed under the following categories:

- (i) those based on critical state concept (Granta Gravel)
- (ii) those based on Rowe's stress-dilatancy theory
- (iii) those incorporating the experimental behaviour of sands in addition to the flow rules as used in any one of the above two models.

Nevertheless all these models are based on isotropic stress history only. With this in view the following aspects are studied:

- (1) Stress-strain behaviour of anisotropically consolidated loose sand along a variety of stress-paths under drained condition.
- (2) Comparison of the experimental and predicted stress-strain behaviour of anisotropically consolidated

loose sand by modified Cam-Clay model.

- (3) The applicability of semi empirical model proposed by Mathur (1975) and Jain (1979) to predict the stress-strain behaviour of anisotropically consolidated loose sand.

CHAPTER 3

DETAILS OF EXPERIMENTAL SET UP AND TESTING PROGRAMME

3.1 General

For investigating field behaviour of soil it is essential to carry out in-situ testing of soil or to obtain undisturbed samples from the field and test them in the laboratory or to prepare remoulded samples of the representative material obtained from the field and test them in the laboratory. The material under investigation in this case being Ganga sand which is cohesionless material, it was not possible to obtain undisturbed samples. Hence remoulded sand samples were tested to (a) obtain strength-deformation characteristics and (b) check the applicability of stress-strain model proposed and developed by Mathur (1975) and Jain (1979) for normally and lightly overconsolidated clays.

3.2 Material Used

Ganga sand which is mostly found in Indo-Gangetic plain, was used for testing. The grain size distribution of sand is given in Figure 2 which indicates that it is uniformly graded and about 65% of material is fine sand and silt with sprinkling of mica. The uniformity coefficient (D_{60}/D_{10}) is 1.43. The physical properties of sand are given in Table 1.

3.3 Equipment for Testing

3.3.1 Triaxial Cell

Standard Norwegian type triaxial cells fitted with frictionless piston assembly with rotating type bushings were used. The cells have 38.1 mm diameter base with two outlets to measure volume changes and pore water pressure.

3.3.2 Volume Gauge

In order to measure small changes in the volume of specimen, U-shaped volume gauge made from uniform thin bore thickwalled glass tube were used. The least count of volume gauge was 0.0084 cc. Entrapped air bubble was used as an indicator in the volume gauge. The gauge was mounted on wooden board and kept horizontal while in use.

3.3.3 Self Compensating Mercury Control System

This system was used to apply and maintain cell and back pressures to specimen during the test. The pressure of water in the triaxial cell, results from the difference in the level between the mercury surfaces in two small cylinders connected by a thin flexible pressure tube as described by Bishop and Henkel (1962).

3.4 Preparation of Specimen

It was desired to investigate the strength-deformation properties of loose sand. Generally reproducible

remoulded clay sample are obtained by resedimenting the clay slurry, but it was found that it was not possible to use this technique for obtaining reproducible remoulded sand specimen. In order to obtain reproducible sand samples the following method which was found to be satisfactory, was adopted.

Based on the physical properties of sand and a few trial samples it was decided to use 128 gms of dry sand for preparation of 38.1 mm diameter and 76.2 mm high samples which would give dry density of 1.475 gms/cc and void ratio of 0.8223. 28 cc of water was used for preparation of specimen which gives about 72% of degree of saturation. It was not possible to use more water as it oozed out of the specimen during compaction. In order to obtain uniform density and compaction, the sand mixed with water, was poured in the former in five equal parts, each layer being lightly compacted with 6 mm diameter rod and 25 blows with about 4 to 6 cms free fall. Before putting in the next layer of sand, the previous layer was scrapped thoroughly in order to avoid layering and to obtain a uniform specimen. A few of the samples thus prepared, were tested to check the uniformity of moisture content along the length of the sample and it was observed that the variation in moisture content of sand taken from three different places (top, middle and bottom of sample) was less than 5%. The detailed procedure for preparation and setting of specimen on triaxial base

is shown in Plates 1 and 2. It can be seen from Plate 1 that split former in two halves with arrangement to create small suction was used for preparing the specimen.

3.5 Setting up of Specimen

Before setting up the specimen all the pressure lines connected to the triaxial cell were flushed with deaired water to expel the entrapped air. A thin rubber tube piece was stretched around the pedestal to provide housing for porous stone and to avoid puncturing of the thin membrane covering the specimen and also to have tight seal by 'O' ring against the leakage through the sides of the pedestal. Boiled porous stone with a filter paper was placed over the pedestal. The specimen was formed on the triaxial base with the help of split former as described above (Refer Plate 1). During the preparation of specimen a small suction pressure was applied in order to keep the membrane stretched along the sides of the former. The required quantity of sand mixed with water which was arrived at on the basis of the trial samples, was poured and the procedure as outlined above, was meticulously followed so as to get reproducible uniform samples. The perspex cylindrical loading cap with a filter paper was kept at the top of the specimen. The split former was then slowly removed. Additional rubber membrane was stretched around the specimen to avoid the possibility of leakage due to puncturing of the first membrane while

mounting the sample. Two 'O' rings were used at each end of the specimen in order to seal the membrane with end platens. Triaxial cell was then assembled and filled with water. The top few centimeters of cell were filled with transformer oil to reduce the leakage and provide lubrication between the piston and the bush. The complete assembly with the sample mounted is shown in Plate 3.

Since the degree of saturation of the specimen at the time of placing was about 72% it was found necessary to circulate water through the specimen to increase the degree of saturation. For this purpose a small cell pressure of 0.2 Kg/cm^2 (3 psi) was applied and water was circulated through the specimen at 0.10 Kg/cm^2 (1.5 psi). A small aperture was made in the loading cap and water along with dissolved air came out of the cell through the aperture in the loading cap. The detailed arrangement is shown in the Plate 4. Once the air bubble stopped coming out of the specimen as observed from the outlet tube circulation of water was stopped and it was plugged.

3.6 Back Pressure Saturation

To ensure complete saturation and to overcome difficulties in measuring negative pore pressure the back pressure saturation method was employed. A small air bubble was entrapped in the volume gauge and the cell and back pressures were built up in stages at slow rate to avoid the

development of pore water pressure and consequent liquifaction of the specimen. Furthermore, the cell pressure was always kept slightly more than the applied back pressure. Back pressure of 1 Kg/cm^2 was applied through the U-tube volume gauge in all the experiment. The cell pressure was maintained at 1.14 Kg/cm^2 during saturation so that there was a small effective alround pressure. The specimen was left for saturation for one day. The back pressure and cell pressure were maintained at constant level with the help of self compensating mercury control system (Bishop and Henkel type). From the observations made on the movement of entrapped air bubble in the U-tube volume gauge it was found that one day was sufficient to completely saturate the sample.

3.7 Consolidation

A few samples were isotropically consolidated to establish void ratio and mean effective stress relationship (e vs. $\ln P'$) in order to calculate λ and κ which are required for prediction of stress-strain behaviour by modified Cam-Clay model. In addition, K_o -consolidation was also carried out in oedometer apparatus to check the variation of λ with pressure.

3.8 Anisotropic Consolidation

The specimen were anisotropically consolidated along the stress ratio q/P' (η) = 0.5. Keeping in view the limitations of the equipment used like pressure gauge and self

compensating mercury control system and the state of specimen it was decided to anisotropically consolidate all the specimen upto mean effective pressure of 2.5 Kg/cm^2 .

Anisotropic consolidation along $\eta = 0.5$ was achieved by applying vertical stress to the specimen in addition to cell pressure during the consolidation. Cell pressure and vertical stress were applied in small increments to avoid significant pore pressure gradient. The time required for anisotropic consolidation was about 4 to 5 hours. The following equation was used to calculate the dead load increments for corresponding cell pressure σ_3 to produce anisotropic consolidation along $\eta = 0.5$ line. The same equation was used in shear process by using different value at every stage of incrementing the load.

$$(\sigma_1' - \sigma_3')a = W + W_h + W_r - a_r \sigma_3$$

also
$$\frac{(\sigma_1' - \sigma_3')}{(\sigma_1' + 2\sigma_3')/3} = \eta$$

substituting we get

$$W = \frac{3\eta}{3-\eta} a(\sigma_3 - u) - (W_h + W_r - a_r \sigma_3)$$

where a = average area of the specimen

a_r = area of piston

W_h = weight of loading frame including hanger

W_r = weight of piston

W = dead load to be added

σ_3 = cell pressure applied

u = back pressure applied to specimen

σ'_3 = effective allround pressure = $\sigma_3 - u$

σ'_1 = effective major principal stress = $\sigma'_3 + \sigma'_v$

3.9 Details of Drained Tests and Various Stress Paths Used

The drained stress controlled tests have been conducted on anisotropically normally consolidated samples along $P' = \frac{\sigma'_1 + 2 \sigma'_3}{3} = \text{Constant}$ with $q = \sigma'_1 - \sigma'_3$ increasing as well as decreasing, $q = \text{Constant}$ with P' increasing and decreasing. One test was conducted on isotropically consolidated sample with $P' = \text{Constant}$ and q increasing. Further additional tests along stress paths A, C and C_1 as shown in Figure 3 were conducted on anisotropically consolidated sand samples. The tests along A, C and $P' = \text{Constant}$ were conducted upto failure in order to establish failure envelope for the material used. The load increments/decrements were applied every one hour. This was decided on the basis of C_v value which was $3.24 \times 10^{-1} \text{ cm}^2/\text{sec}$. Axial strains were measured from the dial gauge mounted on the loading frame hanger. The least count for the dial gauge was 0.01 mm. Volume changes were measured from U-tube volume gauge whose least count was 0.0084 cc.

CHAPTER 4

PRESENTATION OF EXPERIMENTAL DATA

4.1 General

In this chapter stress-strain behaviour of anisotropically consolidated specimen tested along various stress paths have been presented. In addition the test results obtained from isotropic consolidation and K_o -Consolidation in oedometer have also been presented.

4.2 Relationship Between Void Ratio and Mean Effective Pressure

The results for isotropic and anisotropic consolidation tests have been plotted in void ratio and mean effective pressure (e vs. $\ln P'$) space. The average isotropic and anisotropic line for $\eta = 0.5$ are shown in Figure 4. The values of λ (compression index) and κ (swelling index) obtained from isotropic consolidation and swelling test are given in Figure 5. The K_o -consolidation test carried out in oedometer apparatus, has been plotted in Figure 6 which shows that the value of λ is not unique but increases with increase in vertical stress. However, the value of κ does not depend upon the vertical stress as observed from the different unloading line. The variation of λ with pressure (vertical stress) has been plotted in Figure 7. The values of λ as obtained from isotropic consolidation

test for different vertical pressure have also been plotted in Figure 7.

4.3 Results of Drained Tests

4.3.1 P' -constant (q -increasing) Test

P' -constant tests were performed on specimen which were anisotropically consolidated along $e = 0.5$ line. The test results have been plotted in shear stress-volumetric strain and shear stress-shear strain space as shown in Figure 8. One of the P' -constant test was conducted upto failure to obtain the value of q_f . The hyperbolic stress-strain relationship as suggested by Kondner (1963), has been plotted in Figure 9. The value of $(\sigma_1 - \sigma_3)_{ult.}$ has been obtained from inverse of b where b is slope of line in hyperbolic stress-strain plot.

To study the effect of stress history, one P' -constant test was conducted starting from isotropic consolidation line for $P' = 2.5 \text{ Kg/cm}^2$. The stress strain plot is given in Figure 10. In both the tests it is observed that volume decrease (positive volumetric strain) is accompanied by positive shear strain.

4.3.2 P' -constant (q -decreasing) Test

The initial starting point for this test was the same as that for P' -constant (q -increasing) test. The test was conducted upto the isotropic line. During the test it

was observed that volume change from $\eta = 0.5$ to isotropic line ($\eta = 0$) was almost negligible (0.0084 cc which is equal to the least count of volume gauge). The stress-strain characteristics (shear stress vs. shear strain) are plotted in Figure 11. In this test the shear strain is negative.

4.3.3 q-constant (P' -increasing) Test

q-constant tests were performed on anisotropically consolidated samples ($\eta = 0.5$) starting with $P' = 2.5 \text{ Kg/cm}^2$. The stress-strain response of samples are presented in Figure 12. This is a consolidation test with constant shear stress. During this test large volume change (volume decrease) takes place and is accompanied by positive shear strain.

4.3.4 q-constant (P' -decreasing) Test

The initial starting point for this test was the same as for q-constant loading test. The stress-strain behaviour is presented in Figure 13. In this test volume increase (negative volumetric strain) is accompanied by positive shear strain.

q-constant loading as well as unloading tests starting from $P' = 2.5 \text{ Kg/cm}^2$ have also been plotted in void ratio vs. $\ln P'$ space as shown in Figure 4.

4.3.5 Stress Path A

It is conventional drained test in which the lateral stress is kept constant and axial stress is increased. The stress-strain response of the sample along this stress path is shown in Figure 14. Here positive volumetric strain is accompanied by positive shear strain. The test was conducted upto failure to establish the failure envelope. The results of this test have been used for the comparison of strains predicted by various models.

4.3.6 Stress Path C

This test lies in the second quadrant as shown in Figure 3. In this case the lateral pressure is decreased and axial stress is increased. The stress-strain behaviour of specimen along this stress path is shown in Figure 15 which indicates that negative volumetric strain are accompanied by positive shear strain. This test was conducted upto failure to obtain the failure envelope. This test has also been used to study the predictability of strains by various models.

4.3.7 Stress Path C_1

This test lies in fourth quadrant as marked in Figure 3 and in this case the lateral stress increase is accompanied by decrease in axial stress. With the testing equipment available in the laboratory it was not possible

to conduct the test below the isotropic line (i.e. extension test) as such this test was conducted starting from $n = 0.5$ and terminating close to isotropic line. The stress-strain characteristics of sand specimen tested along C_1 path are presented in Figure 16 which indicates that positive volumetric strain is accompanied by positive shear strain initially, and then increment in shear strain become negative. This test has also been used to compare the experimental and predicted stress-strain behaviour using various models.

4.4 Failure Envelope

On the basis of results of drained tests along P' -constant, A-path and C-path, the values of shear stress and mean effective stress at failure for the respective tests are plotted in P' - q space and failure envelope drawn as shown in Figure 17. It is observed that failure envelope for sand is a straight line having the slope (M) equal to 1.3. This value of M is used for all subsequent calculations.

4.5 Critical State Line

The void ratio at failure for A, C and P' -constant stress paths have been plotted in e vs. $\ln P'$ space and critical state line has been established as shown in Figure 4.

CHAPTER 5

PREDICTED AND OBSERVED STRESS
STRAIN BEHAVIOUR AND DISCUSSIONS

5.1 General

In general the deformation response of soil is controlled by the mean stress and shear stress induced by external loading. The theory of elasticity considers the complete decoupling i.e. volumetric response depends only on the mean stress and shear deformation is controlled by shear stress. Many of the models suggested, are following this conception in the pre-yielding region. However, the experimental results reported by researchers indicate the coupled behaviour as emphasized by Scot and Ko (1969). The semi empirical model proposed by Mathur considers this coupled behaviour i.e. deformation response whether it is volumetric or shear, depends on both mean stress and shear stress. Mathur suggested that the stress strain behaviour of soil along any stress path may be predicted from the results from two series of tests:

- (1) Pure shear (P' -constant tests, and
- (2) Consolidation (q -constant) tests.

The assumption, that the effect of any loading increment is made up of the separate effects of the corresponding increase of P' and q , is as follows

$$\delta \epsilon_v = \frac{\partial \epsilon_v}{\partial p'} \delta p' + \frac{\partial \epsilon_v}{\partial q} \delta q$$

$$\delta \epsilon = \frac{\partial \epsilon}{\partial p'} \delta p' + \frac{\partial \epsilon}{\partial q} \delta q$$

where $\delta \epsilon_v$ and $\delta \epsilon$ are changes in volumetric and shear strain respectively due to application of load increment ($\delta p'$, δq).

$\frac{\partial \epsilon_v}{\partial p'}$, $\frac{\partial \epsilon_v}{\partial q}$, $\frac{\partial \epsilon}{\partial p'}$ and $\frac{\partial \epsilon}{\partial q}$ are model parameters to be determined for p' -constant and q -constant tests. The method of determining parameters is given below.

5.2 Determination of Model Parameters

5.2.1 Mathur (1975) and Jain (1979) Model

5.2.1.1 p' -constant (q -increasing) Test

The experimental data for this test are plotted in $\ln(\frac{q_f}{q_f - q})$ vs. ϵ and $(\frac{q}{q_f})$ vs. ϵ_v as shown in Figures 18 and 19. These plots are similar to those suggested by Wroth (1968). The slope of straight line relationships obtained in above plots are related to parameters $\frac{\partial \epsilon}{\partial q}$ and $\frac{\partial \epsilon_v}{\partial q}$ as under

$$\frac{\partial \epsilon}{\partial q} = \frac{D_3}{q_f - q}$$

where $1/D_3$ is the slope of straight line in $\ln(\frac{q_f}{q_f - q})$ vs. ϵ plot, and

$$\frac{\partial \epsilon_v}{\partial q} = \frac{D_1}{q_f}$$

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where $1/D_1$ is slope of straight line in $(\frac{q}{q_f})$ vs. ϵ_v plot. Here q_f is defined as the value of q at failure for a constant P' test. In this case $D_1 = 0.0033$ and $D_3 = 0.008$.

5.2.1.2 P' -constant (q -decreasing) Test

The results of P' -constant (q -decreasing) tests are plotted in the form of $\ln(\frac{2q_f}{q_f + q})$ vs. ϵ as shown in Figure 20. During this test it was observed that the volume change for the stress level under investigation in this study, was of the order of least count (0.0084 cc) of volume measurement device and hence no reliance was given to the data and it is assumed that volume change parameter $\frac{\partial \epsilon_v}{\partial q}$ due to P' -constant (q -decreasing) test is almost equal to zero and hence neglected. The relationship between $\ln(\frac{2q_f}{q_f + q})$ and shear strain ϵ is a straight line with slope $1/C$. The parameters are obtained from

$$\frac{\partial \epsilon}{\partial q} = \frac{C}{q_f + q}$$

and

$$\frac{\partial \epsilon_v}{\partial q} = \frac{B}{q_f}, \text{ with } B = 0 \text{ for sand investigated.}$$

As observed from the Figure 20 the slopes of the plot $\ln(\frac{2q_f}{q_f + q})$ vs. ϵ is changing progressively as the test proceeds towards extension. This has been discussed in detail in later part of this chapter.

5.2.1.3 q-constant (P' -increasing) Test

The results are plotted in volumetric strain ϵ_v vs. $\ln P'$ and shear strain ϵ vs. $\ln P'$ as shown in Figure 21. The plots are straight line with slopes equal to α_1 and α_3 for volumetric and shear strains respectively. The parameters are obtained from

$$\frac{\partial \epsilon_v}{\partial P'} = \frac{\alpha_1}{P'}$$

$$\text{and} \quad \frac{\partial \epsilon}{\partial P'} = \frac{\alpha_3}{P'}$$

where $\alpha_1 = 0.013$ and $\alpha_3 = 0.0040$.

5.2.1.4 q-constant (P' -decreasing) Test

In this test volumetric strains ϵ_v are plotted against $\ln P'$ and shear strains ϵ against $\ln(P' - P'_f)$ as shown in Figures 22 and 23 respectively. The slopes of straight lines are β_1 and β_3 for volumetric and shear strains respectively. The parameters are obtained from the relationship

$$\frac{\partial \epsilon_v}{\partial P'} = \frac{\beta_1}{P'}$$

$$\text{and} \quad \frac{\partial \epsilon}{\partial P'} = \frac{\beta_3}{(P' - P'_f)}$$

where P'_f is the value of P' at failure in q-constant (P' -decreasing) test and $\beta_1 = 0.007$ and $\beta_3 = -0.00062$.

The volumetric and shear strain increment parameters as obtained from four basic tests are given in Table 3. The parameters are obtained from the average of two tests for each basic test path (P' -constant loading and unloading, and q -constant loading and unloading).

5.2.2 Modified Cam-Clay Model (Burland, 1965)

Besides Mathur's (1975) model, modified Cam-Clay (Burland, 1965) has also been used to predict the stress-strain behaviour of sand. The basic equation for computing incremental volumetric and shear strains are 2.3 and 2.4 as given in Chapter 2. The parameters required for prediction by this model are λ , κ and M . λ and κ are computed from e vs. $\ln P'$ plotted in Figure 5 for isotropic consolidation and swelling test. The value of λ is not constant and increases with increase in pressure while κ was constant as observed from Figure 6. The variation of λ with vertical pressure has been shown in Figure 7. It may be due to the fact that the mica present in the sample might be getting crushed at higher pressure. Furthermore, it was observed that the value of λ showed some variation even for the same pressure in isotropic consolidation test. This may be attributed to variation of mica content in different samples. The average isotropic consolidation line and average anisotropic consolidation line for $n = 0.5$ are shown in Figure 4. The average value of $\lambda = 0.017$

corresponding to $P' = 2.5 \text{ Kg/cm}^2$ and $\kappa = 0.0068$ are used for prediction by modified Cam-Clay model. The value of $M = 1.3$ has been obtained from the failure envelope and used for prediction by this model.

5.3 Predicted and Observed Stress-Strain Behaviour Along Various Stress Paths

Volumetric and shear strains are calculated using Mathur (1975) and Jain (1979) model and modified Cam-Clay model. For calculation of strains a stress path is broken into a number of small probes of $\delta P'$ and δq .

For using Mathur and Jain model the parameters obtained from appropriate P' -constant and q -constant tests are used to calculate volumetric and shear strains due to probe along the selected stress path. The following equations are used in the computations of volumetric and shear strain in the respective quadrants as indicated in Figure 3.

$$\delta \epsilon_v = \frac{D_1}{q_f} \delta q + \frac{\alpha_1}{P'} \delta P'$$

For stress path in first quadrant

$$\delta \epsilon = \frac{D_3}{(q_f - q)} \delta q + \frac{\alpha_3}{P'} \delta P'$$

$$\delta \epsilon_v = \frac{D_1}{q_f} \delta q + \frac{\beta_1}{P'} \delta P'$$

For stress path in second quadrant

$$\delta \epsilon = \frac{D_3}{q_f - q} \delta q + \frac{\beta_3}{(P' - P'_f)} \delta P'$$

$$\delta \epsilon_v = \frac{B}{q_f} \delta q + \frac{\beta_1}{P'} \delta P'$$

For stress path in
third quadrant

$$\delta \epsilon = \frac{C}{q_f + q} \delta q + \frac{\beta_3}{(P' - P'_f)} \delta P'$$

$$\delta \epsilon_v = \frac{B}{q_f} \delta q + \frac{\alpha_1}{P'} \delta P'$$

For stress path in
fourth quadrant

$$\delta \epsilon = \frac{C}{q_f + q} \delta q + \frac{\alpha_3}{P'} \delta P'$$

In the above equations P' and q are current values and q_f is the failure value of q corresponding to the current value of P' . The summation of strain increments caused by each stress probe gives the total strain.

For modified Cam-Clay model equations 2.3 and 2.4 are used for calculating incremental volumetric and shear strain. In these equations η , P' and e are the current values. Appropriate sign is used while taking the values of η and $\delta P'$.

5.4 Comparison of Experimental and Predicted Results

5.4.1 Stress Path A

Experimental and predicted stress strain behaviour is shown in Figure 24. As observed from the plot the volumetric and shear strain are predicted fairly accurately by Mathur and Jain's model. (The observed variation, particularly in case of volumetric strain, may not be that

significant considering the fact that A-test path data is for one test only). It can also be seen from the Figure 24 that modified Cam-Clay predicts well the volumetric strain while it underpredicts the shear strain considerably. This may be attributed due to the fact that there is no unique flow rule for sands as indicated in Figure 29.

5.4.2 Stress Path C

Experimental and predicted stress-strain behaviour are shown in Figure 25. Once again Mathur and Jain's model predicts volumetric and shear strain reasonably well and scatter is well within the experimental limitations as explained in preceding paragraphs. Modified Cam-Clay model predicts volumetric strain close to experimental values at lower stress ratio and underpredicts at higher stress ratio. The shear strain is again very much underpredicted.

5.4.3 Stress Path C_1

This path lies in fourth quadrant and parameters for prediction of strains are determined from P' -constant (q -decreasing) and q -constant (P' -increasing) basic tests. As pointed out earlier under 'determination of parameters' that during P' -constant (q -decreasing) test there was negligible volume change hence the volumetric strain parameter B was assumed to be zero. Besides the shear strain parameter C was determined on the basis of test taken upto isotropic line. From the trend of plot

$\ln\left(\frac{2q_f}{q_f + q}\right)$ vs. ϵ it appears that the value of C might have been higher, had the test been taken below the isotropic line i.e. extension test. Since the extension test could not be carried out in the laboratory, it is not fair to rely heavily on the value of C thus calculated. However, an attempt has been made to predict C_1 -path by taking the shear strain parameter for P' -constant (q -decreasing) test as (1) equal to shear strain parameter ' D_3 ' for P' -constant (q -increasing) test and (2) 80% of D_3 . The predicted and experimental volumetric and shear strains vs. n plots are shown in Figure 26. The volumetric strain is predicted reasonably well whereas experimental values of shear strain are closer to predictions made with C equal to 80% of D_3 at higher stress ratio n , while at lower values of n experimental values are closer to prediction made with C equal to D_3 . Further tests are to be done to verify these findings and then decide upon the value of C to be adopted for the model, however, it is tentatively suggested that C may be taken equal to D_3 with $B = 0$. For this path, the predictions of volumetric and shear strain have also been made with modified Cam-Clay model and shown in Figure 26. The volumetric strain is underpredicted throughout the stress path whereas the shear strains are overpredicted at lower values of n i.e. near the isotropic line.

Besides the stress-strain behaviour of P' -constant (both anisotropic and isotropic) and q -constant stress paths

have been predicted by modified Cam-Clay model and shown in Figures 8, 10 and 27. It can be seen from these figures that it underpredicts volumetric and shear strains for both P' -constant and q -constant tests.

5.5 Observed Deformation Behaviour of Sand

Tatsuoka (1972) studied the deformation behaviour of sand under triaxial condition. The tests were conducted on isotropically consolidated sand samples. On q/P' , P' plane he plotted the points of equal shear strain for different stress paths and found that equal shear strain points for different stress paths lie on one curve. In other words shear strain is unique for a combination of values of q and P' independent of stress path (see Figure 28 inset). He called these lines as 'equi γ lines'. In the present study which has been done on anisotropically consolidated samples, the points of equal shear strain obtained from different stress paths, have been plotted in q/P' , P' plane as suggested by Tatsuoka and shown in Figure 28. It is evident from this figure that equi γ lines obtained by testing anisotropically consolidated samples are very much similar to those obtained by Tatsuoka. The shear strains plotted in Figure 28 are increments with reference to initial anisotropic consolidated state. However, it is found that the shear strains measured for isotropic consolidation and anisotropic consolidation to reach the pre-shear state corresponding to $\eta = 0.5$ and $P' = 2.5 \text{ Kg/cm}^2$ are not the

same. (The shear strain for anisotropically consolidated sample corresponding to $\eta = 0.5$ and $P' = 2.5 \text{ Kg/cm}^2$ is 0.59% while for isotropically consolidated sample it is 0.21% (see Figure 28)). This clearly shows that the shear strain for a combination of q/P' and P' depends on the stress history of the sample before shearing and this relationship is unique only for the increment of shear strain with reference to pre-shear state. Besides this relationship is not unique and these lines are to be established for every granular soil.

Tatsuoka (1972) showed that the relationship between q/P' and $\partial \epsilon_v^P / \partial \epsilon^P$ is dependent on stress path and this relationship is unique only when the state of sample reaches peak (see inset in Figure 29). The relationship between q/P' and $\partial \epsilon_v / \partial \epsilon$ has been plotted for different stress paths in Figure 29 which shows close similarity with the findings of Tatsuoka. This may explain the inapplicability of flow rule $\partial \epsilon^P / \partial \epsilon_v^P = \frac{2\eta}{M^2 - \eta^2}$ as used in modified Cam-Clay model, for sands. Hence the prediction of shear strains by this model are not comparable with the experimental results as shown in Figures 3, 10, 24, 25, 26 and 27. Furthermore, it is seen from the Figure 29 that the lines corresponding to various stress paths when extended, meet at a single point. This point corresponds to the strain increment ratio $\partial \epsilon_v / \partial \epsilon$ equal to zero which represents the critical state of the sample. From this figure it is

evident that the stress ratio corresponding to this critical state is 1.3 which is same as the value of M obtained from the failure envelope. This method can be usefully employed to determine the value of M in case of constant stress tests on granular soils where the value of deviatoric stress at failure cannot be accurately determined.

Similar to equi γ lines as established by Tatsuoka (1972), points of equal volumetric strain with reference to the pre-shear state obtained from different stress paths, have been plotted on q/P' and P' plane. It is observed that these points of equal volumetric strain lie on a single curve as shown in Figure 30. In other words the volumetric strain is unique for a combination of values of q/P' and P' independent of stress path. However, this needs to be further investigated by carrying out the test along the stress paths in the third quadrant (i.e. P' , q both decreasing) since in this investigation stress paths in other three quadrants have been studied. Moreover, it is also observed from this figure that volumetric strain changes sign from positive to negative (volume decrease to swelling) about the P' -constant stress path (slightly to its left).

Oda (1972) studied the influence of initial fabrics on the mechanical properties of sand and its effect on deformation behaviour. He defined fabric as the spatial arrangement of solid particles and associated voids. He used resin to fix the structure of samples and

defined the ratio S_z/S_x as the ratio of principal planes in an ellipsoidal surface formed out of all the contact surfaces gathered and stuck together in unit volume of sand. This ratio represents fabric characteristics of sample. He conducted the conventional drained test (A-path) on such samples and found that S_z/S_x is linearly connected with stress ratio σ_1/σ_3 and dilatancy rate $\partial \epsilon_v / \partial \epsilon_1$. He established the relationship between this ratio σ_1/σ_3 and strain increment ratio $\partial \epsilon_v / \partial \epsilon_1$ as

$$\frac{\sigma_1}{\sigma_3} = K_5 \frac{\partial \epsilon_v}{\partial \epsilon_1} + K_6$$

which is the equation of straight line. K_5 and K_6 are constants. His experimental data fitted well with the equation proposed by him. He concluded that linear relationship between σ_1/σ_3 , $-(\partial \epsilon_v / \partial \epsilon_1)$ and S_z/S_x are fundamental in granular mechanics.

During his investigation only A-path was considered and other paths were not investigated. In order to compare Oda's findings with the experimental results the stress ratio σ_1/σ_3 and strain increment ratio $-(\partial \epsilon_v / \partial \epsilon_1)$ have been plotted for different stress paths as shown in Figure 31. This figure confirms the findings of Oda (1972) for A stress path and it can also be seen that similar straight line relationship exists for other stress paths investigated. Furthermore, all these lines corresponding to different stress paths merge at the peak. The ratio of

$\frac{\sigma_1}{\sigma_3}$ at peak is 3.28 which corresponds with the value of $M = 1.3$ obtained from the failure envelope. Incidentally such a plot could be usefully employed to precisely determine the failure stress in case of constant stress tests.

Based on the experimental results Tatsuoka and Ishihara (1974) pointed out that the family of successive yield loci are curved in P' - q space but nevertheless open in the direction of P' -axis. They also reported that the shear strains as measured in tests with various stress paths hardly depend on the stress-path and shear-strain contour resemble the successive yield loci. However, in their investigation all the tests are conducted on isotropically consolidated samples. In Figure 32 points of equal shear strain measured for various stress paths have been plotted and shear strain contours are drawn. The first of the two conclusion arrived at by Tatsuoka and Ishihara (1974) that shear strain hardly depends on stress-path is conclusively confirmed from this figure. However, the shape of the shear strain contour are more concave downwards for anisotropically consolidated samples. This shows that the shape of yield locus and shear strain contours may not be the same in case of anisotropically consolidated samples.

The volumetric strain contours are also plotted in Figure 32 which indicates that a constant volume line is very much curved instead of being an almost vertical as suggested by Vermeer (1980). Thus it can be emphasized

that stress history predominantly controls the shear and volumetric strain response of granular material. Hence the double hardening model suggested by Vermeer seems to be an oversimplified concept for stress-strain behaviour of sands.

CHAPTER 6

CONCLUSION AND SCOPE FOR FURTHER INVESTIGATION

6.1 Conclusion

In this study attempt has been made to study the deformation behaviour of Ganga sand and compare it with the deformation behaviour of other sands reported by various authors, mostly the Japanese group. Comparisons have been made with the predicted deformation behaviour using various models and experimental behaviour of anisotropically normally consolidated loose sand. The following conclusions are drawn:

- (1) The stress strain behaviour of anisotropically consolidated loose sand shows close similarity with the findings of Tatsuoka and Ishihara (1972) in respect of:
 - (a) unique incremental shear strain contours in $\eta - P'$ stress space being independent of stress path (Figure 28),
 - (b) the relationship between stress ratio η and strain increment ratio $(\frac{\partial \epsilon_v}{\partial \epsilon})$ dependent on stress path and unique only as critical state is approached (Figure 29).
- (2) The experimental results indicate that stress ratio $\frac{\sigma_1}{\sigma_3}$ is linearly related to strain increment ratio $(\frac{\partial \epsilon_v}{\partial \epsilon_1})$ as reported by Oda (1972), the conventional drained test. However, it is observed that the linear relationship is path dependent (Figure 31).

- (3) Modified Cam-Clay model in most of the cases underpredicts the shear strains (Figures 8, 10, 24, 25, 26 and 27). This is due to the fact that there is no unique flow rule for sands as assumed in the model. The dependence of flow rule on stress path has been emphasized by Tatsuoka and Ishihara (1972) (see Figure 29 inset) and confirmed in this investigation (Figure 29).
- (4) The semi-empirical model proposed by Mathur (1975) and Jain (1979) predicts the stress-strain behaviour of sand very close to the actual behaviour for a wide variety of stress-path specially in the range which are of practical importance.

From the findings presented here and the results of Japanese studies it would appear that a semi-empirical approach to modelling the stress path and stress history dependent stress-strain behaviour would be more realistic. This would in fact be similar to the stress path approach to the study of soil behaviour.

6.2 Scope for Further Investigation

In case of normally and lightly overconsolidated clays Jain (1979) has reported that loading parameter D_1 and D_3 increases linearly with increase in initial mean effective pressure P'_i and parameter α_1 and α_3 are independent of P'_i . This needs verification in case of sands by varying the initial mean effective pressure P'_i .

TABLE 1 : Physical Properties of Ganga Sand.

| Specific gravity | e_{min} | e_{max} | e for samples | Relative density | Uniformity coefficient |
|------------------|-----------|-----------|--------------------|------------------|------------------------|
| 2.68 | 0.58 | 1.07 | 0.8223 | 50.1% | 1.43 |

TABLE 2 : Basic Tests Required for Prediction of Stress Paths in Different Quadrants (Refer Figure 3).

| Quadrant | Stress path specification | Basic tests needed for parameter evaluation | |
|----------|---------------------------|---|------------------------|
| I | P' -increasing | q -Constant test | (stress path - E_1) |
| | q -increasing | P' -Constant test | (stress path - B) |
| II | P' -decreasing | q -Constant test | (stress path - E) |
| | q -increasing | P' -Constant test | (stress path - B) |
| III | P' -decreasing | q -Constant test | (stress path - E) |
| | q -decreasing | P' -Constant test | (stress path - B_1) |
| IV | P' -increasing | q -Constant test | (stress path - B_1) |
| | q -decreasing | P' -Constant test | (stress path - E_1) |

TABLE 3 : Model Parameters for Various Basic Tests on Anisotropically Consolidated Loose Sand.

$$p'_1 = 2.5 \text{ Kg/cm}^2 \quad n = 0.5$$

| Basic test | Parameters |
|-----------------------------------|--|
| p' -constant (q -increasing) | $D_1 = 0.0033, \quad D_3 = 0.008$ |
| p' -constant (q -decreasing) | $B = 0, \quad C = D_3 \text{ (proposed)}$ |
| q -constant (p' -increasing) | $\alpha_1 = 0.013, \quad \alpha_3 = 0.0040$ |
| q -constant (p' -decreasing) | $\beta_1 = 0.007, \quad \beta_3 = - 0.00062$ |

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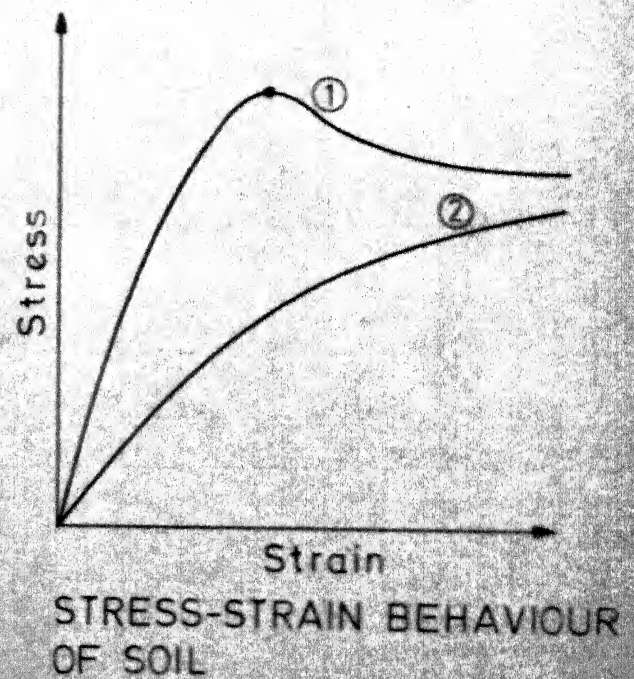
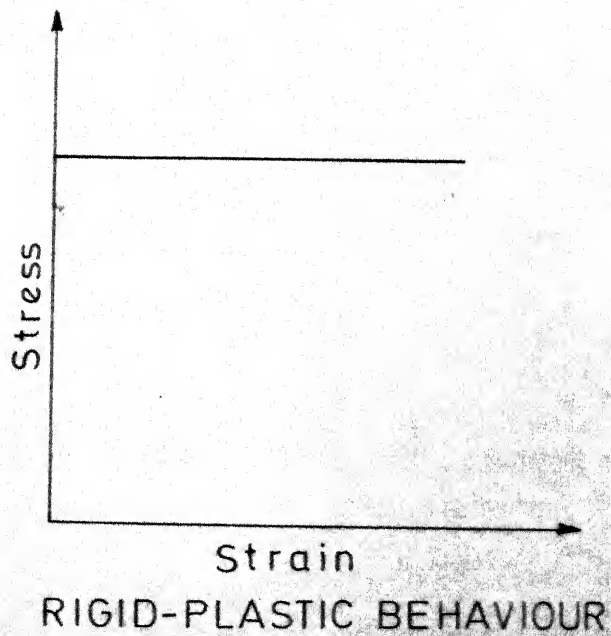
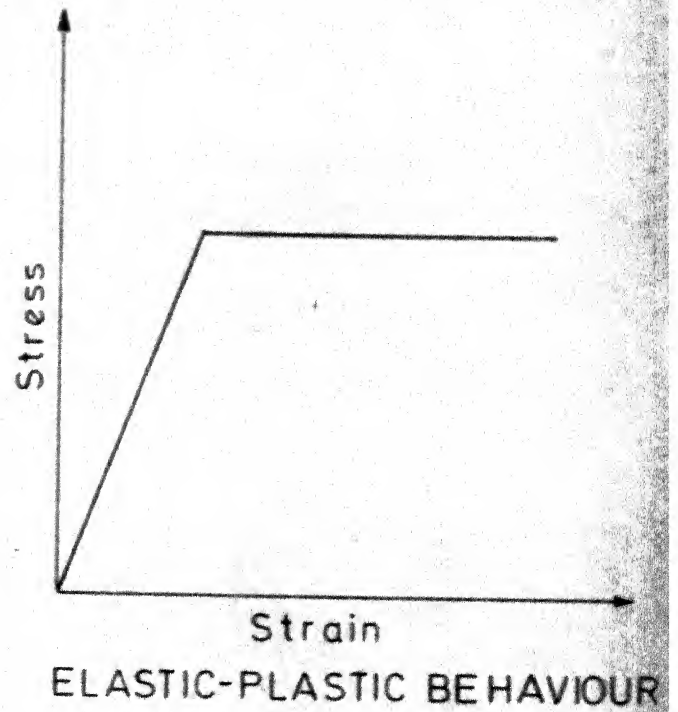
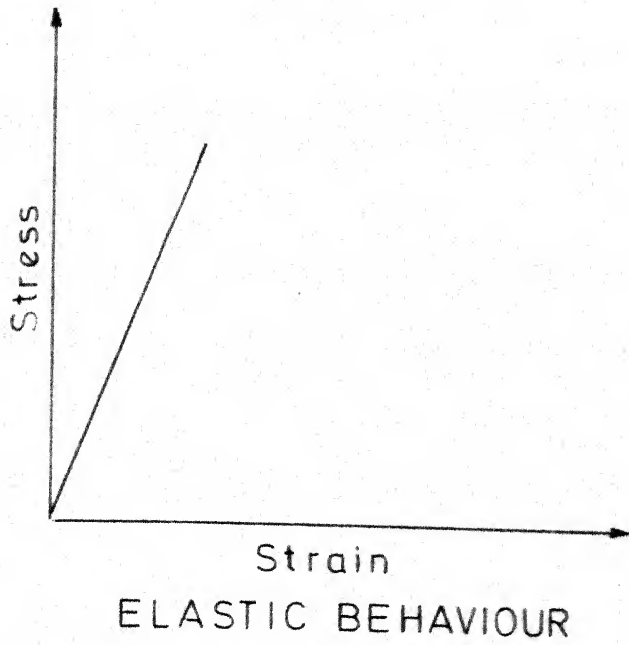


FIG.1 STRESS-STRAIN BEHAVIOUR OF DIFFERENT MATERIALS

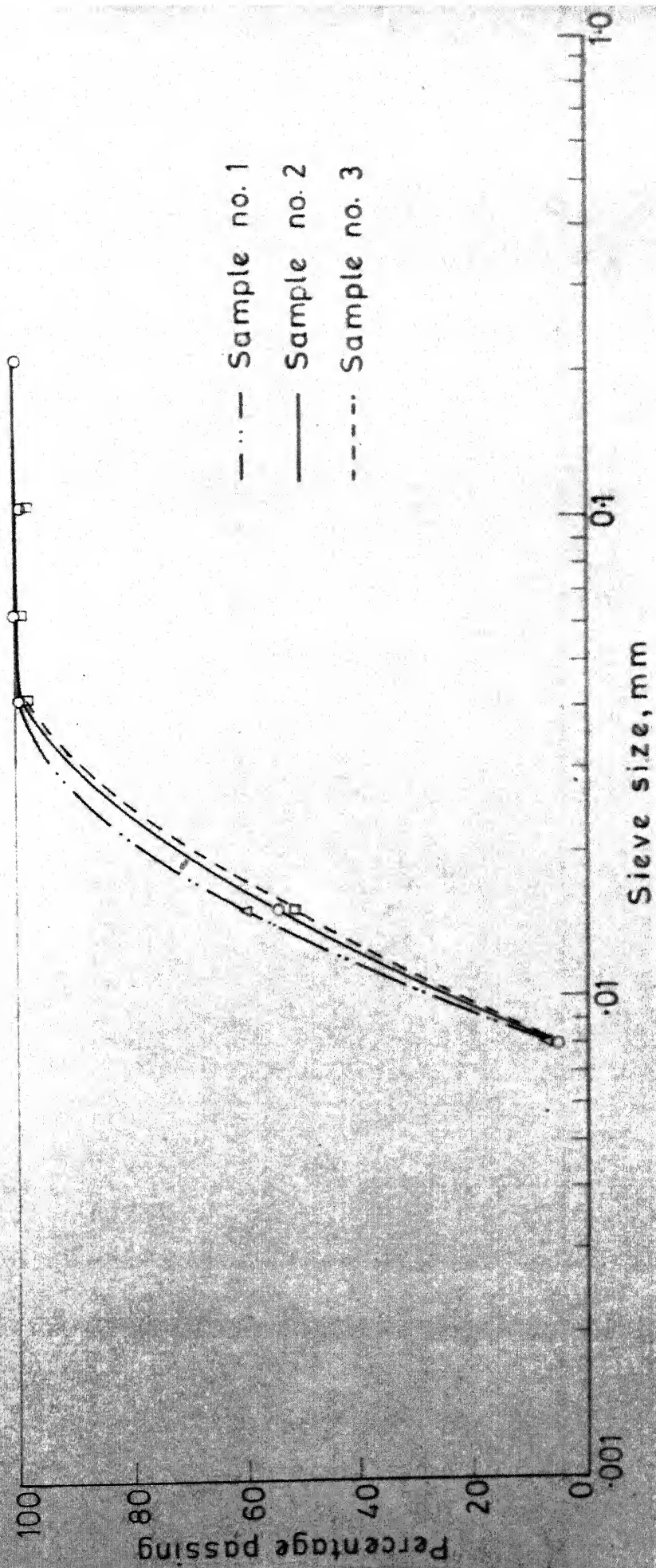


FIG. 2 GRAIN SIZE DISTRIBUTION OF SAND

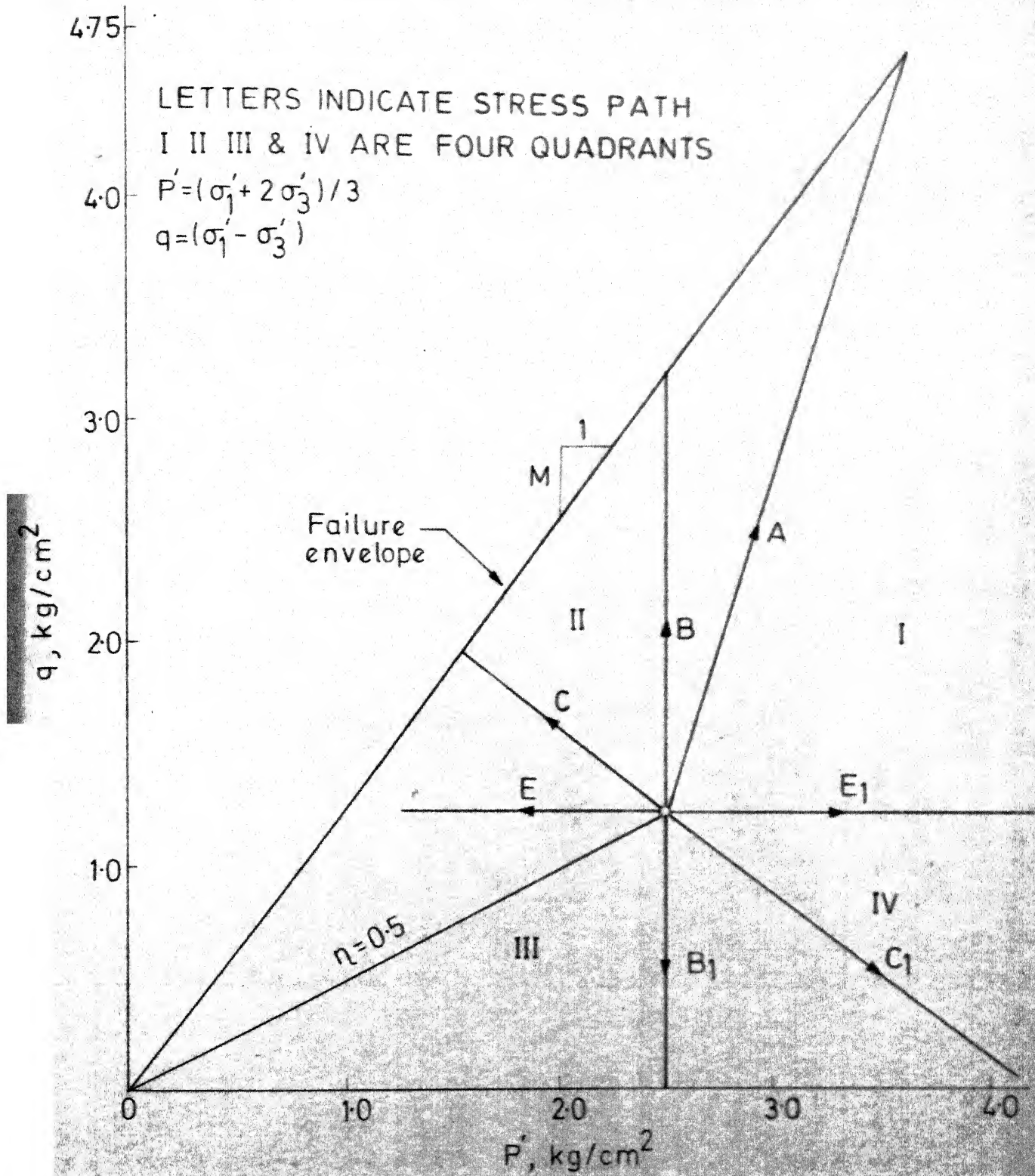


FIG.3 VARIOUS STRESS PATHS FOLLOWED IN THIS INVESTIGATION AND FAILURE ENVELOPE

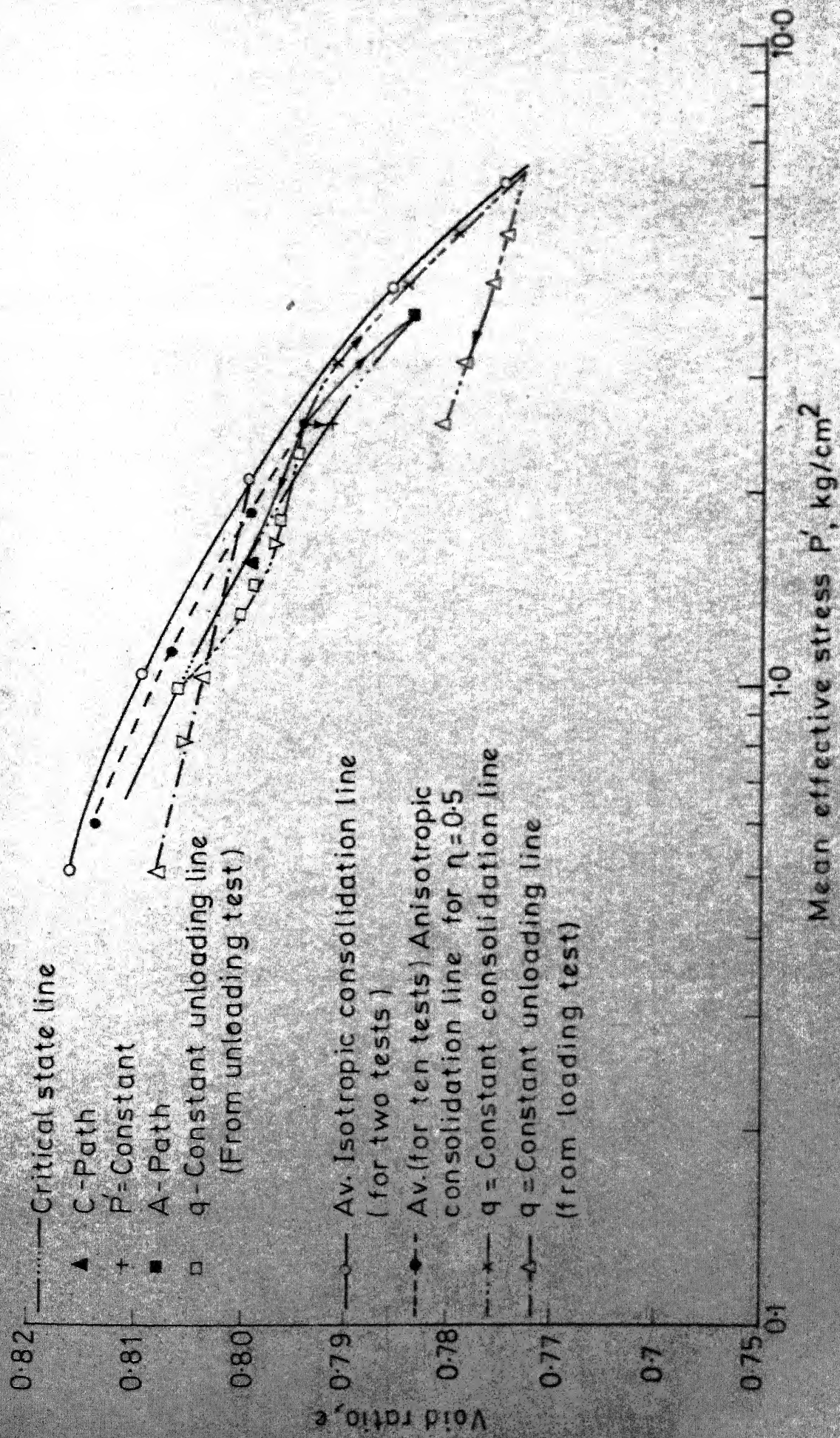


FIG.4 RELATIONSHIP BETWEEN VOID RATIO AND MEAN EFFECTIVE STRESS P' FOR ISOTROPIC, $\eta = 0.5$ AND $q = \text{CONSTANT}$ CONSOLIDATION AND SWELLING

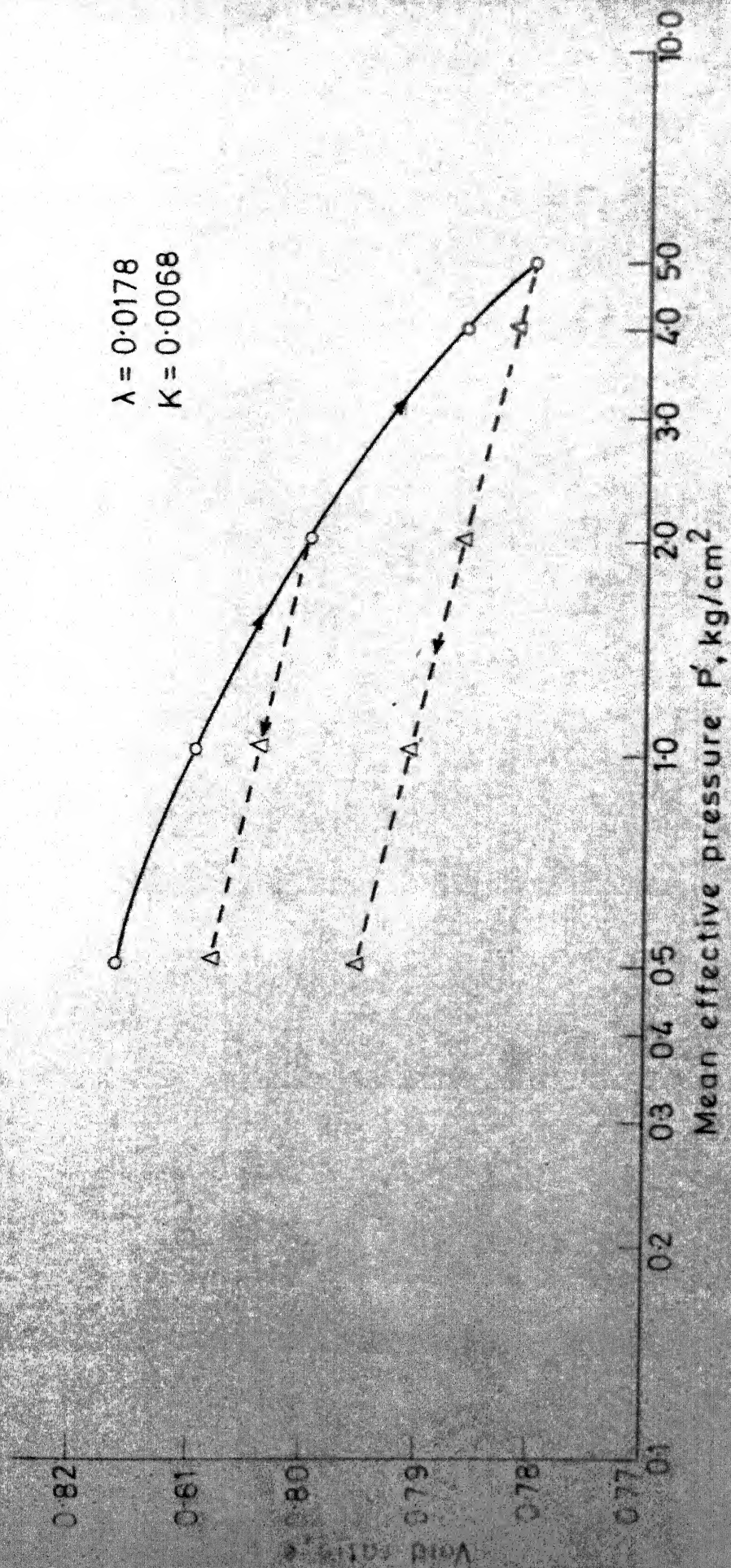


FIG.5 ISOTROPIC CONSOLIDATION AND SWELLING LINE FOR DETERMINATION OF λ AND K FROM TRIAXIAL TEST

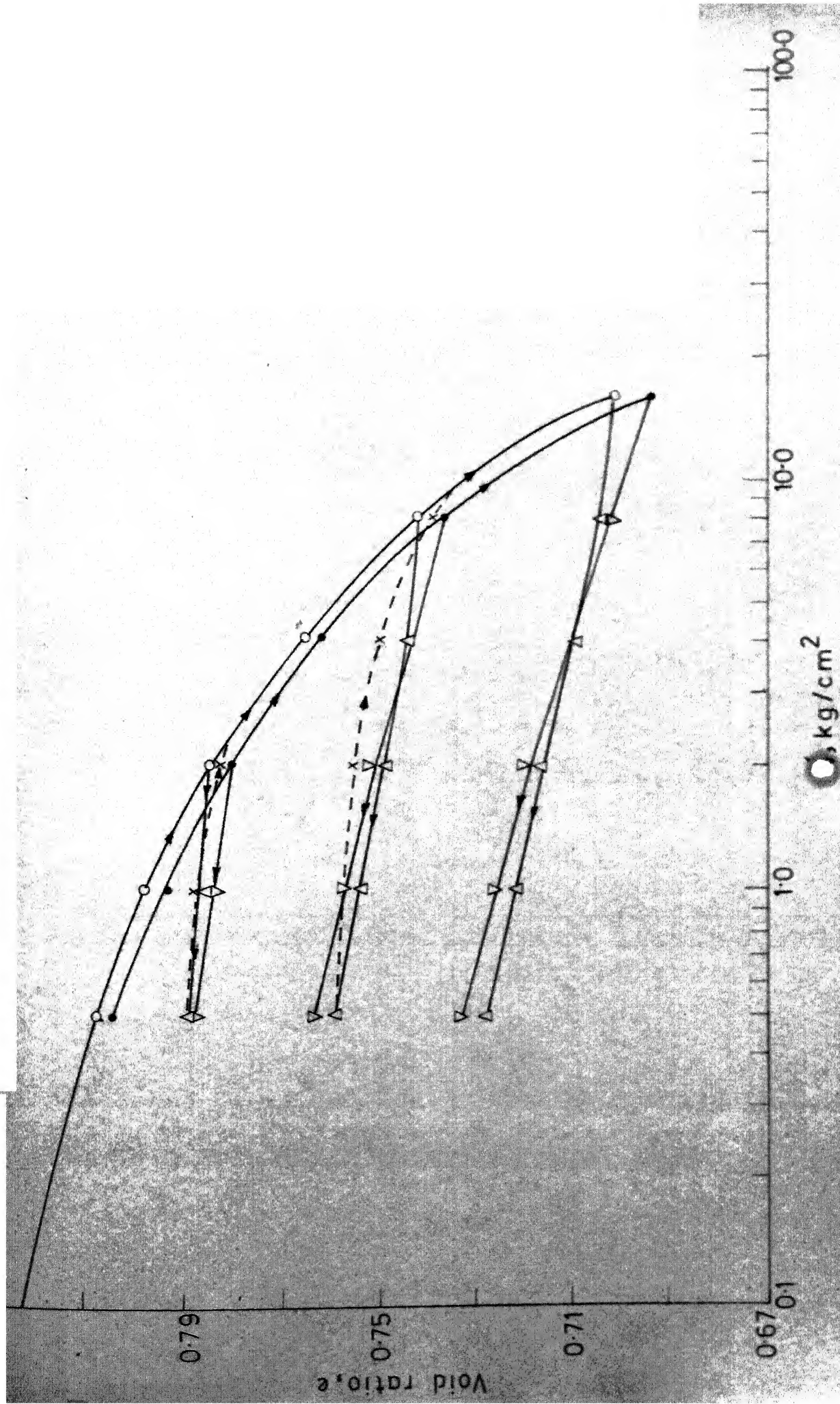


FIG. 6 RELATIONSHIP BETWEEN VOID RATIO AND MEAN EFFECTIVE PRESSURE
AS OBTAINED FROM OEDOMETER TEST

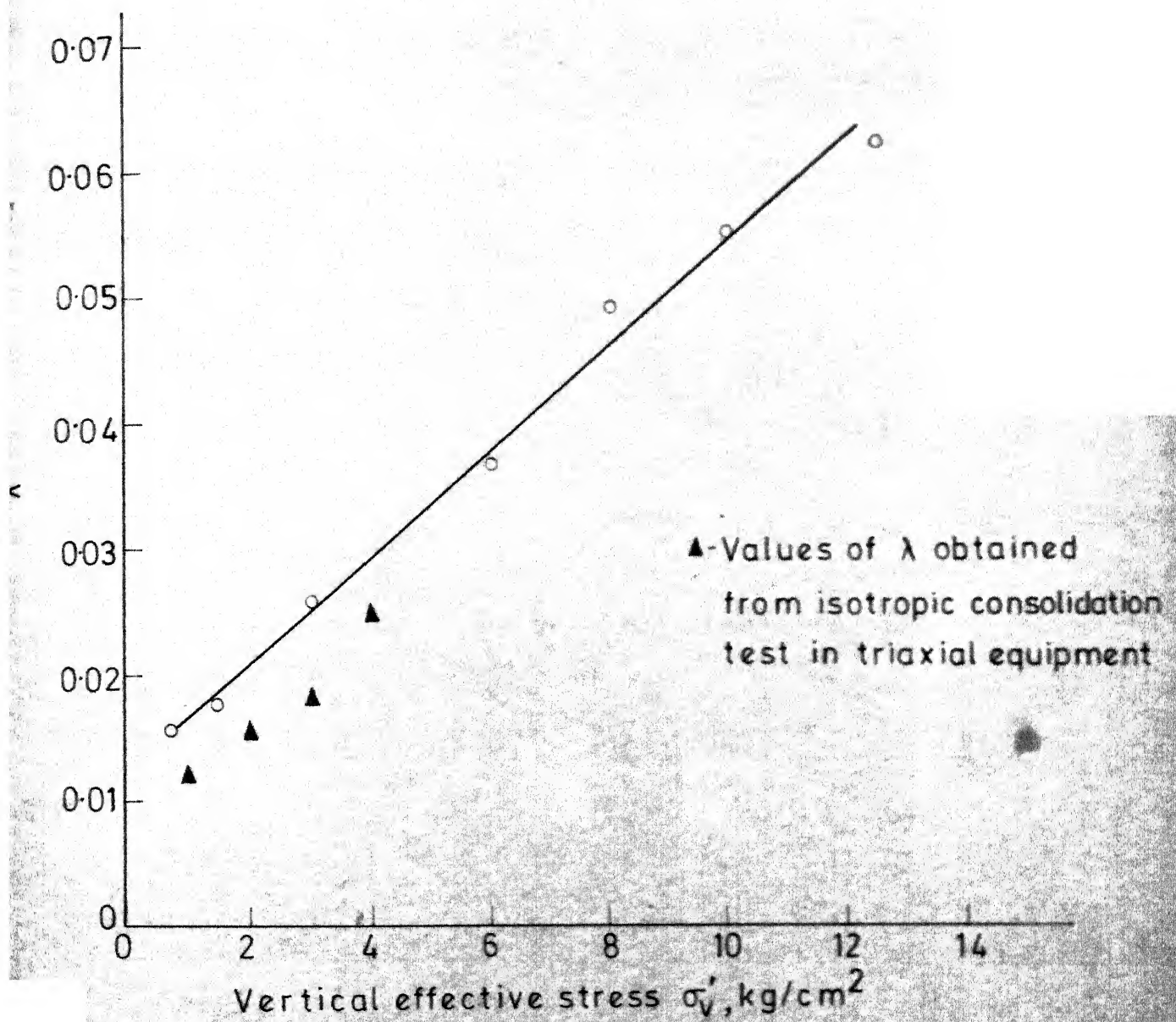


FIG.7 VARIATION OF λ WITH σ'_v AS OBTAINED FROM OEDOMETER CONSOLIDATION TEST

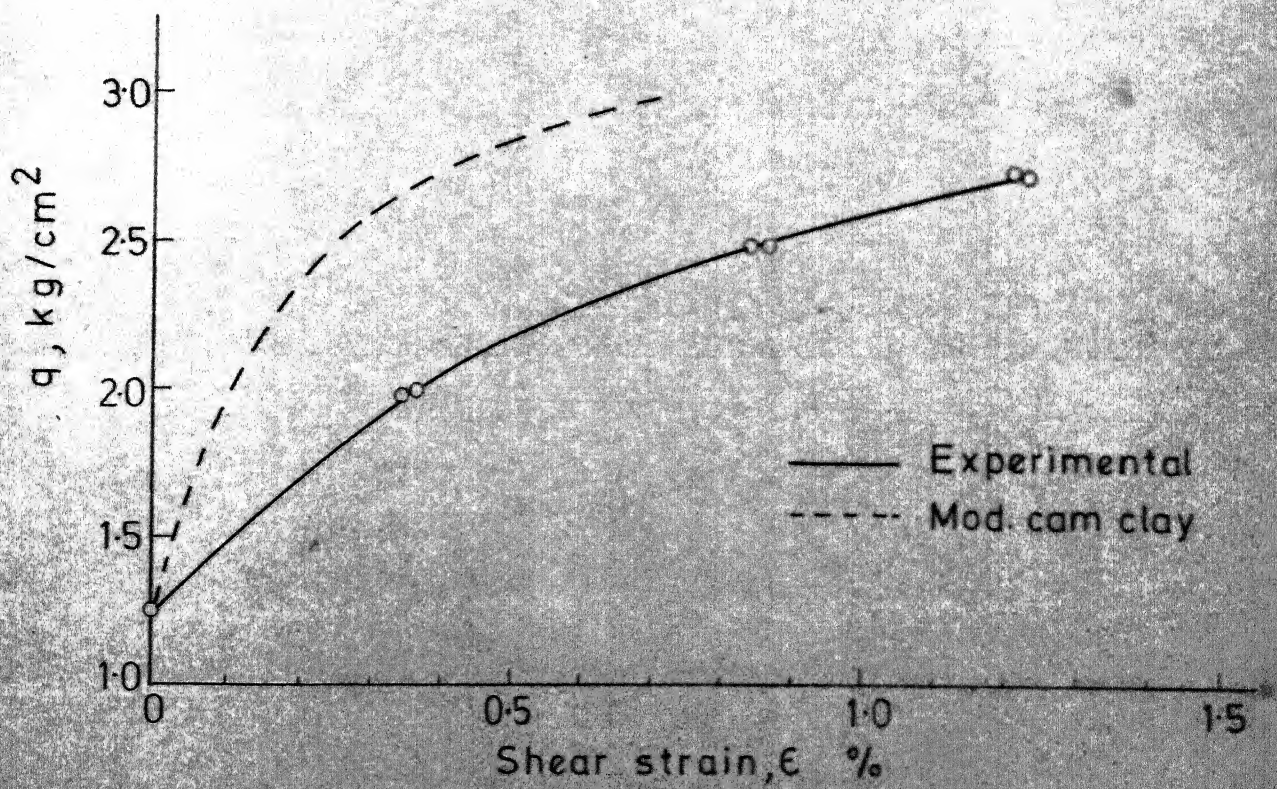
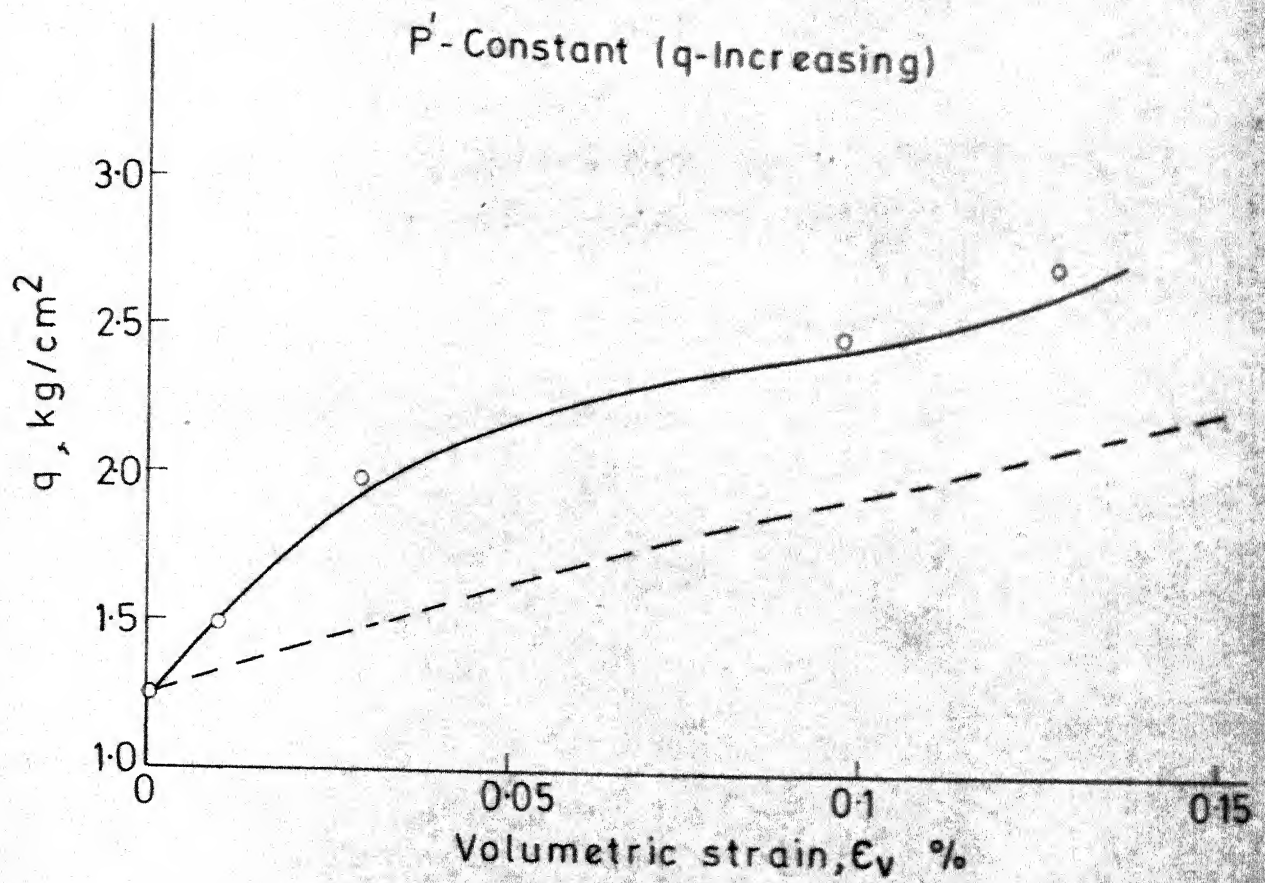


FIG.8 STRESS STRAIN BEHAVIOUR

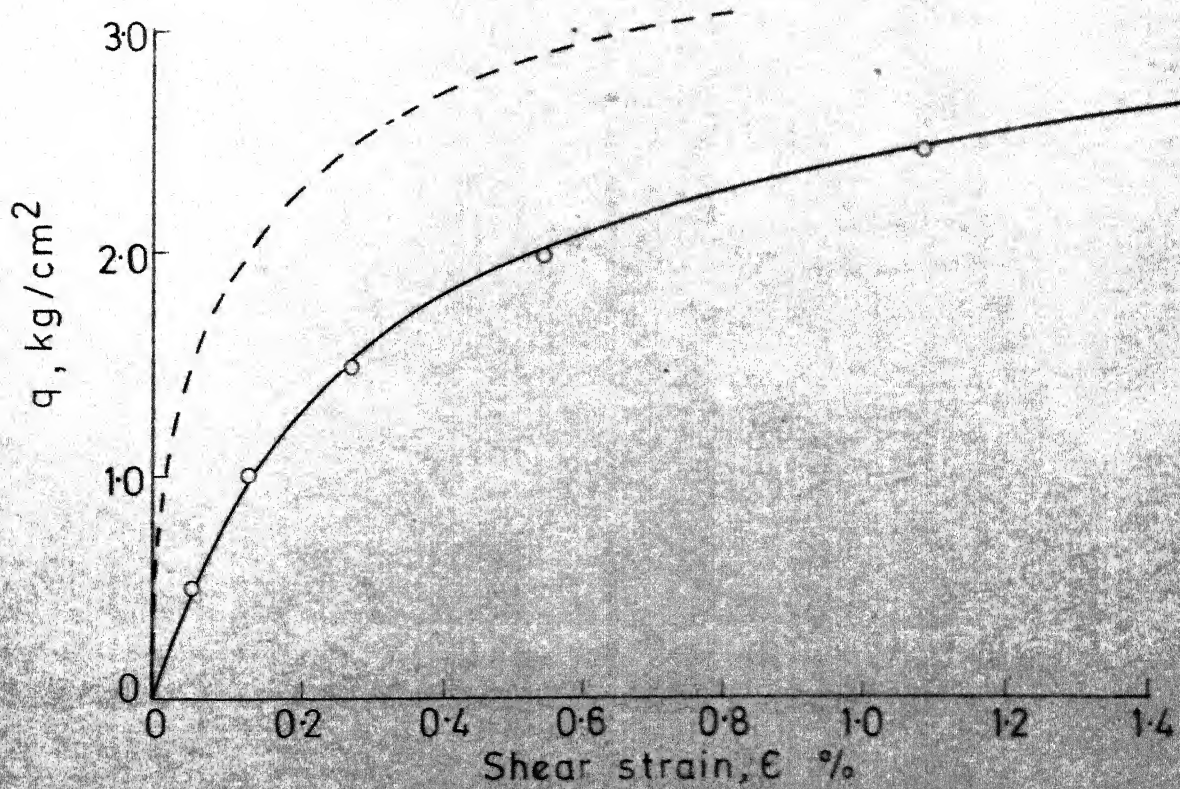
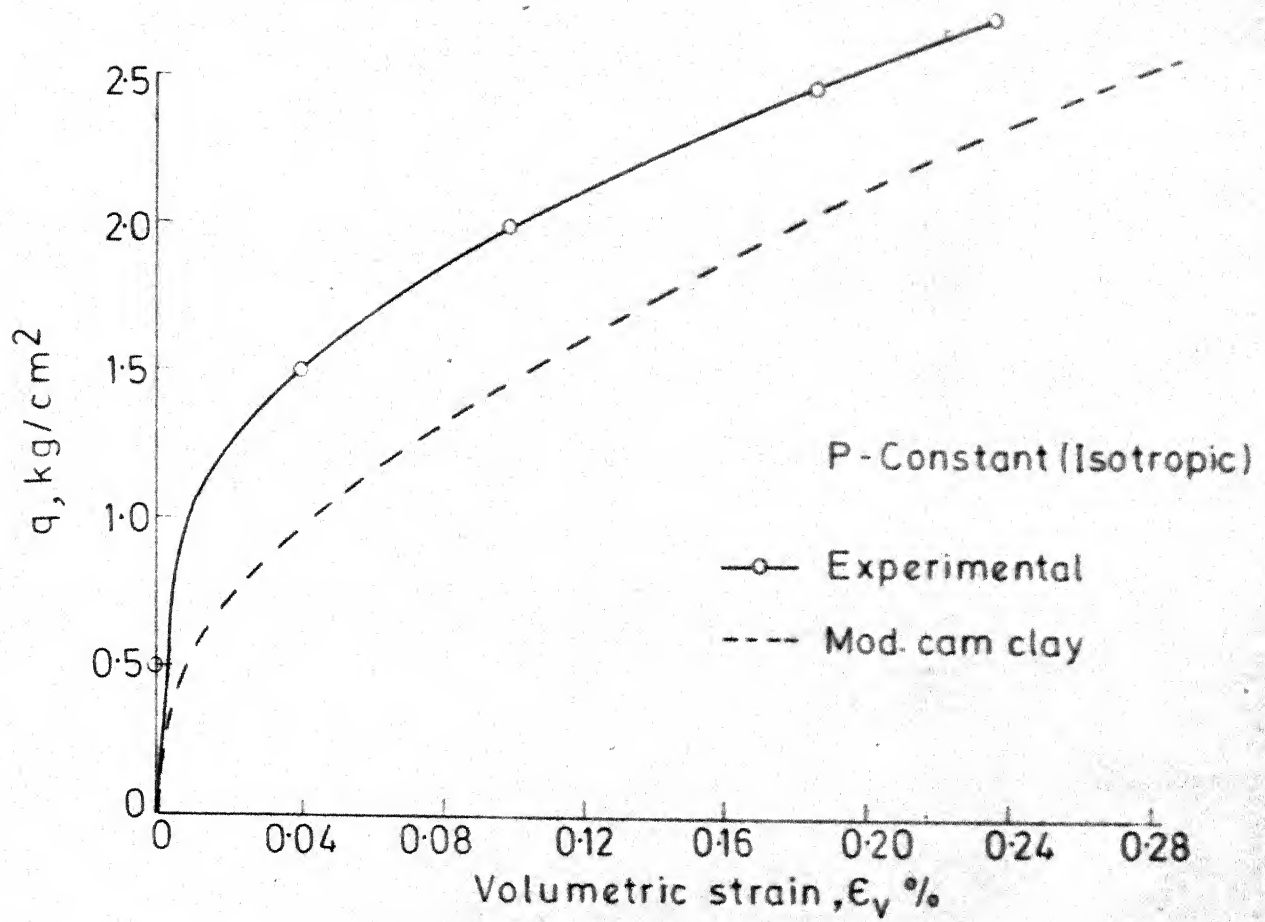


FIG.10 STRESS STRAIN BEHAVIOUR

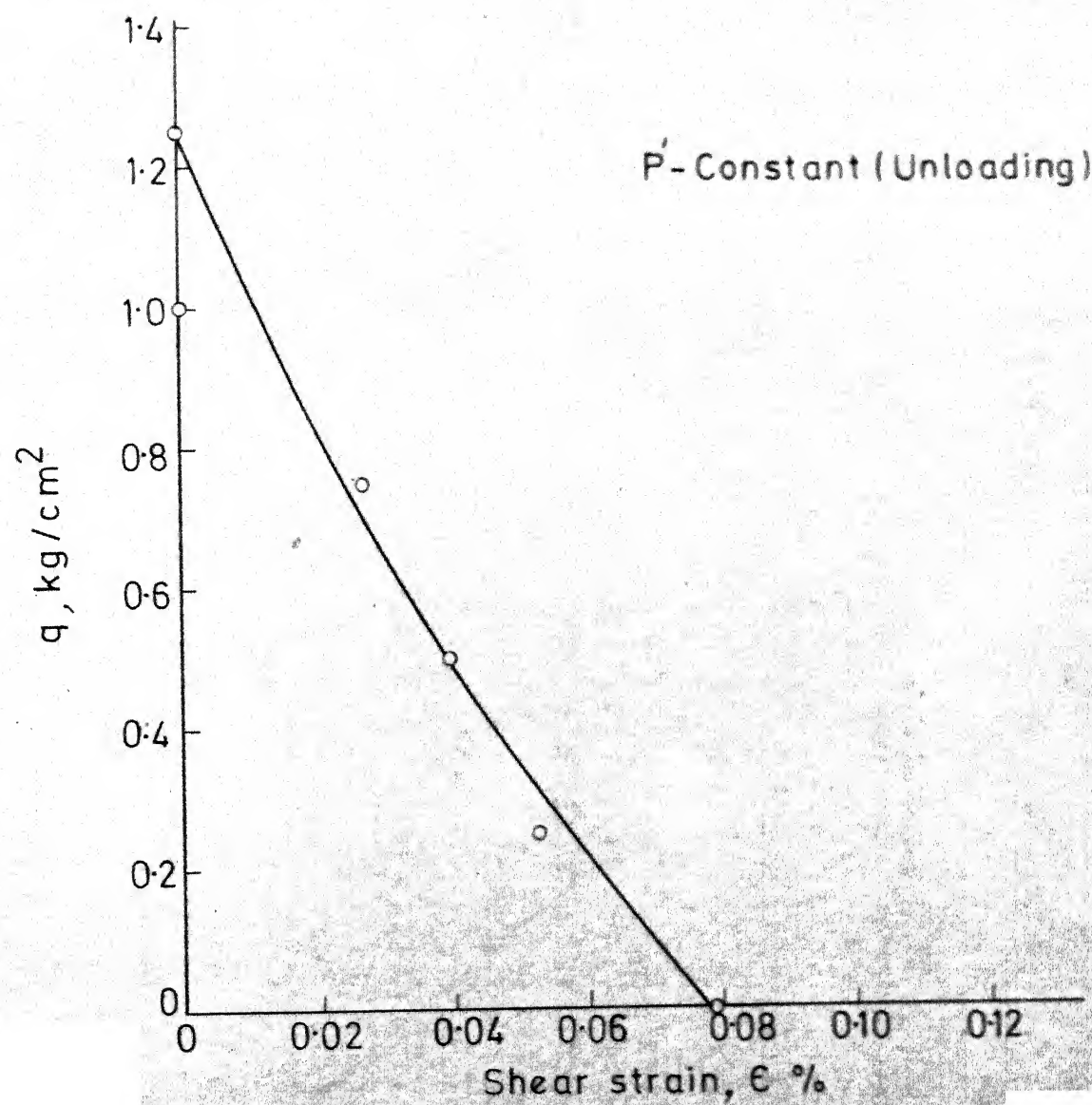


FIG.11 STRESS STRAIN BEHAVIOUR

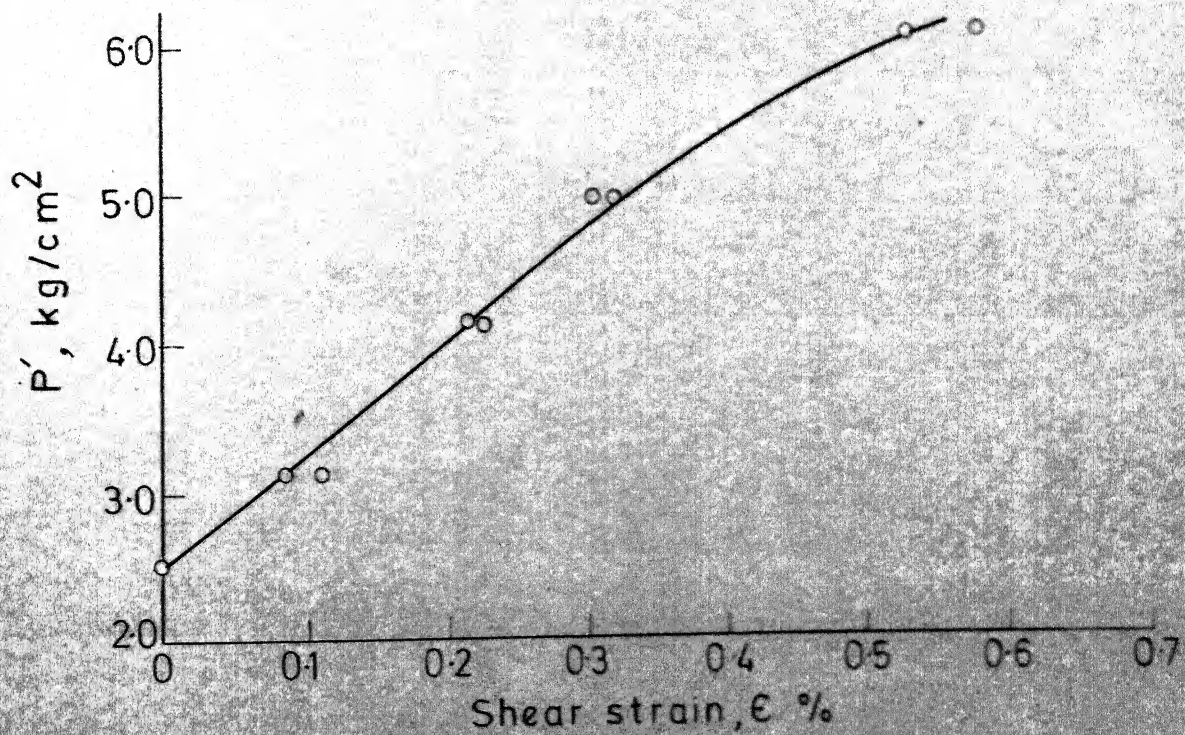
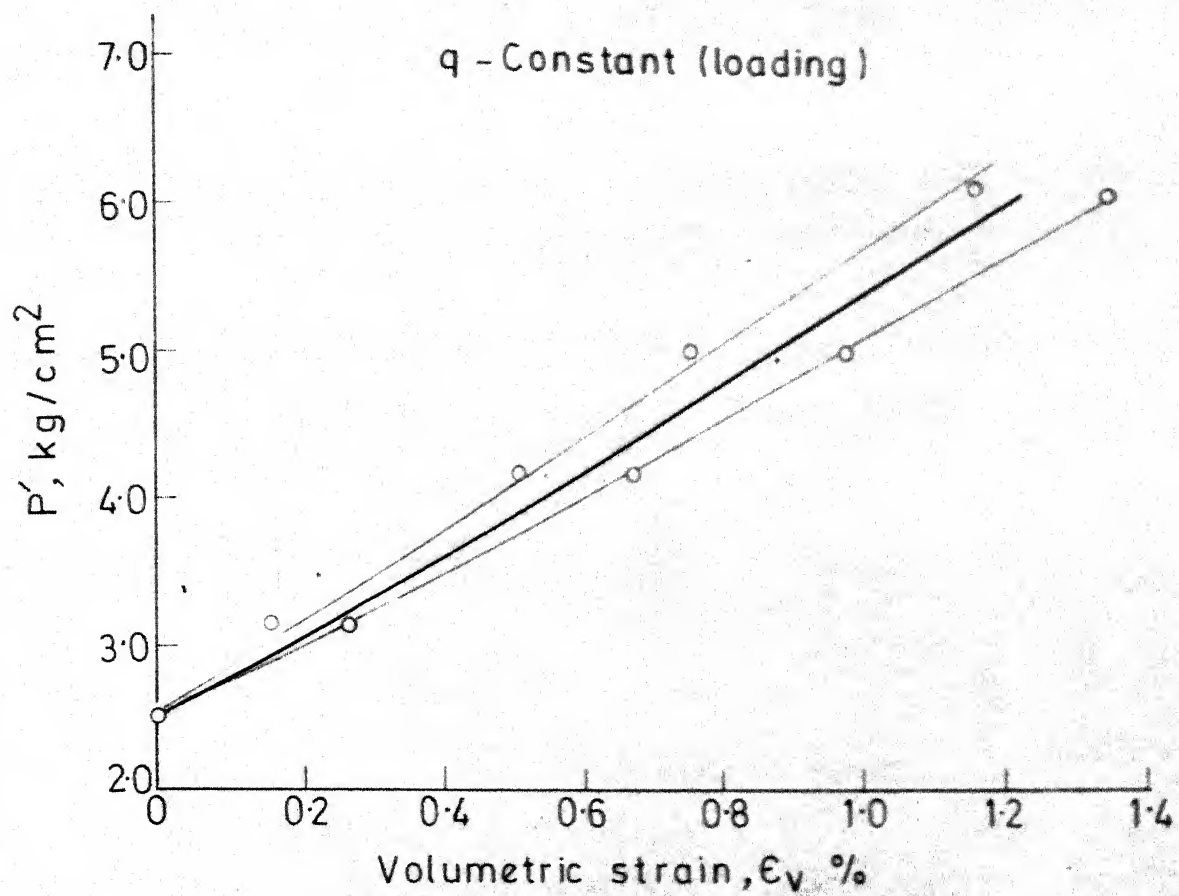


FIG.12 STRESS STRAIN BEHAVIOUR

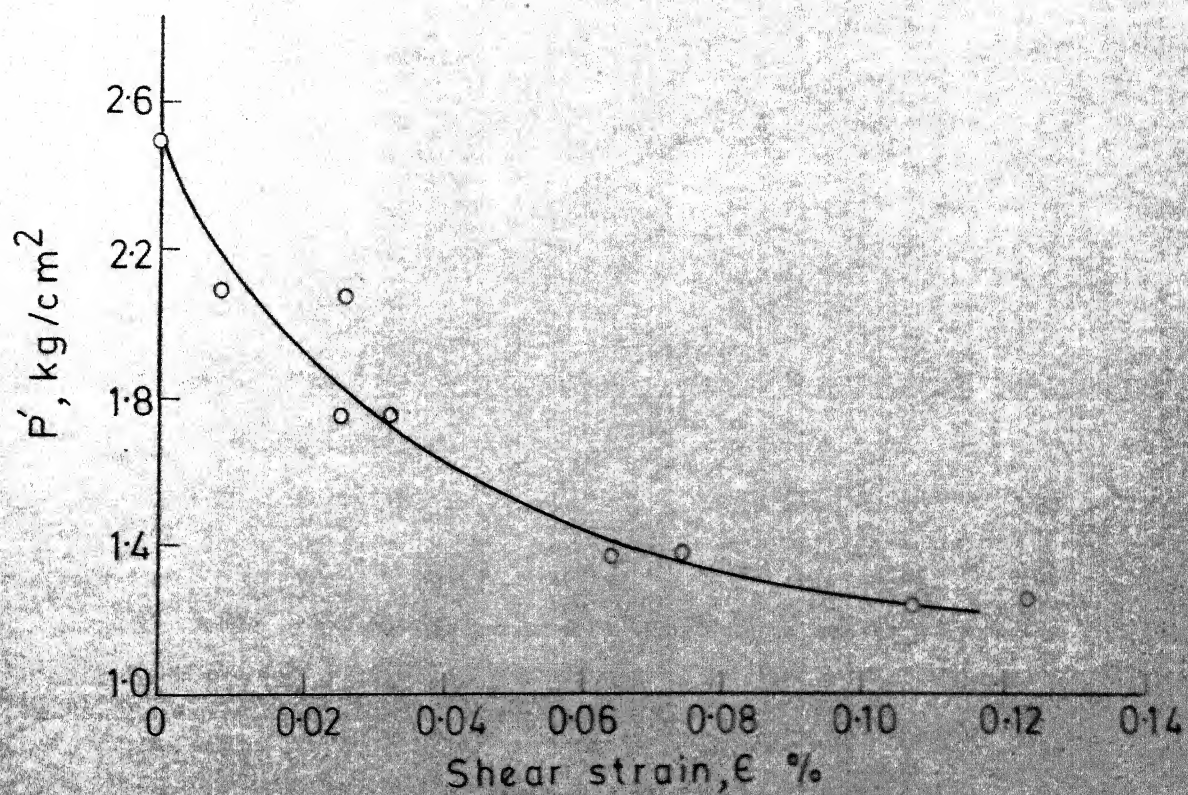
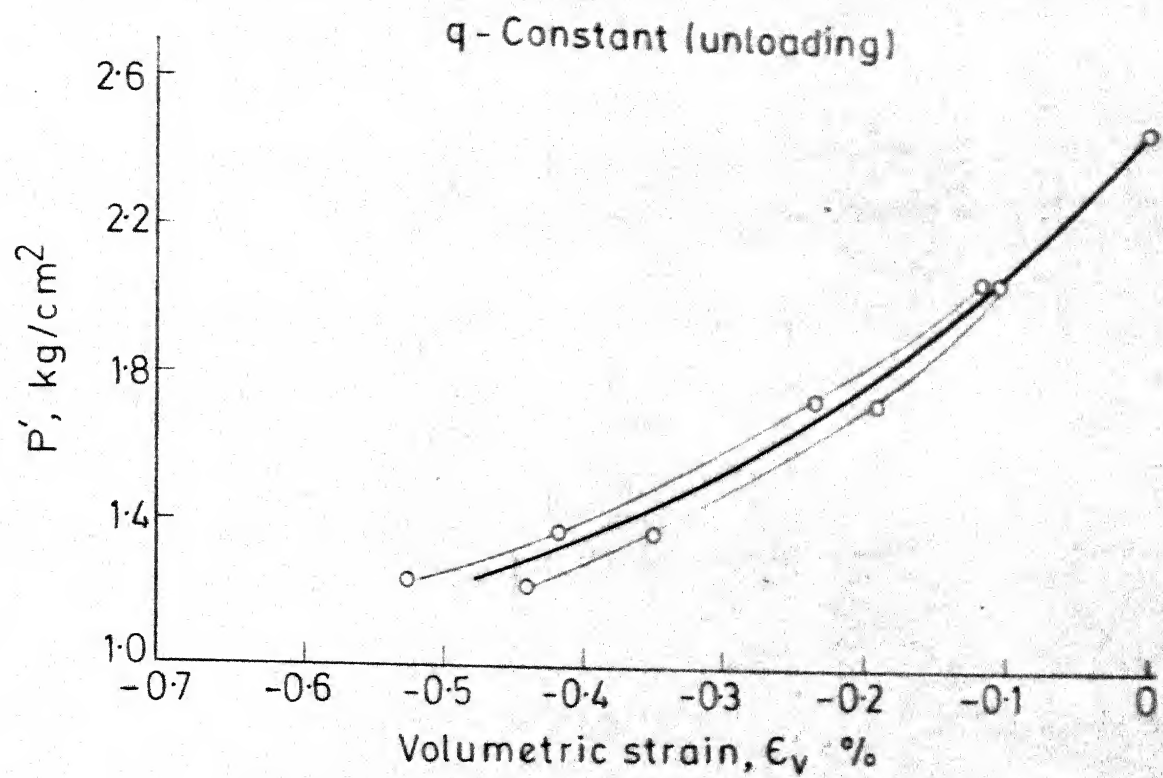


FIG.13 STRESS STRAIN BEHAVIOUR

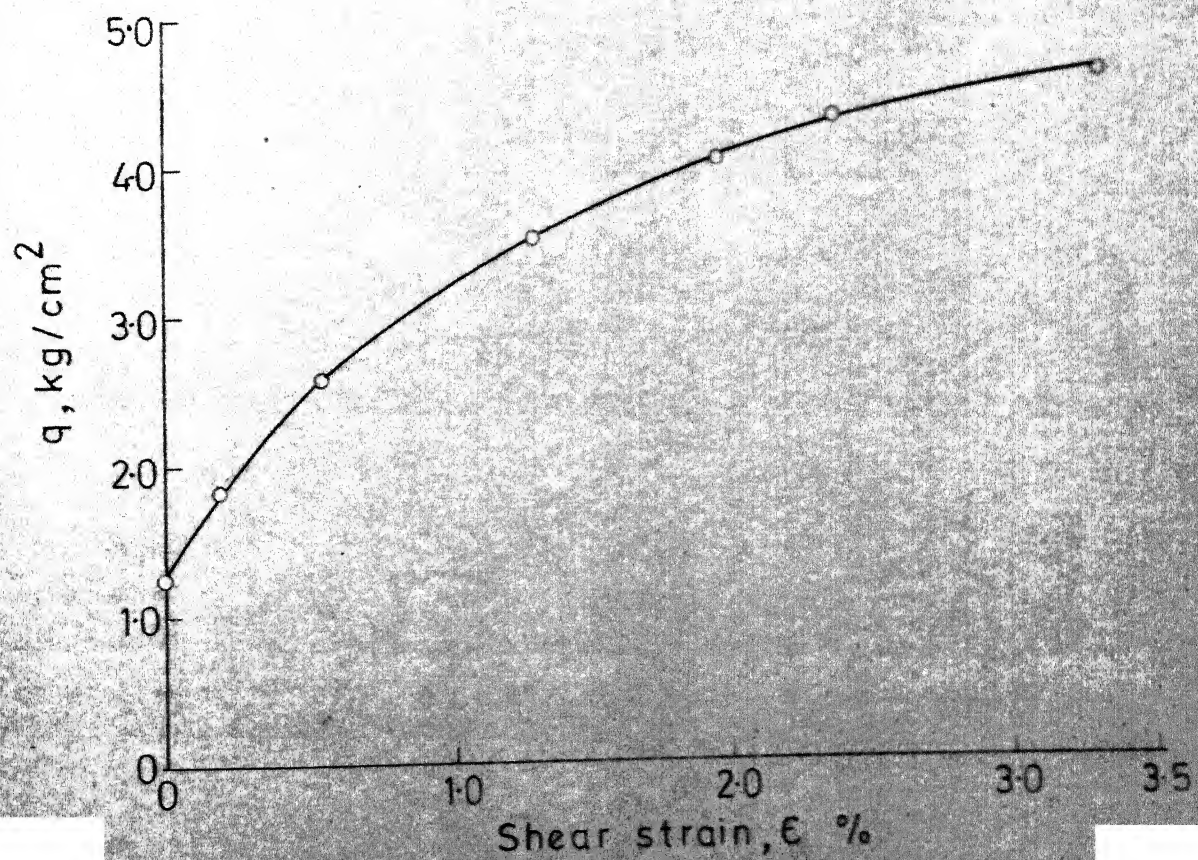
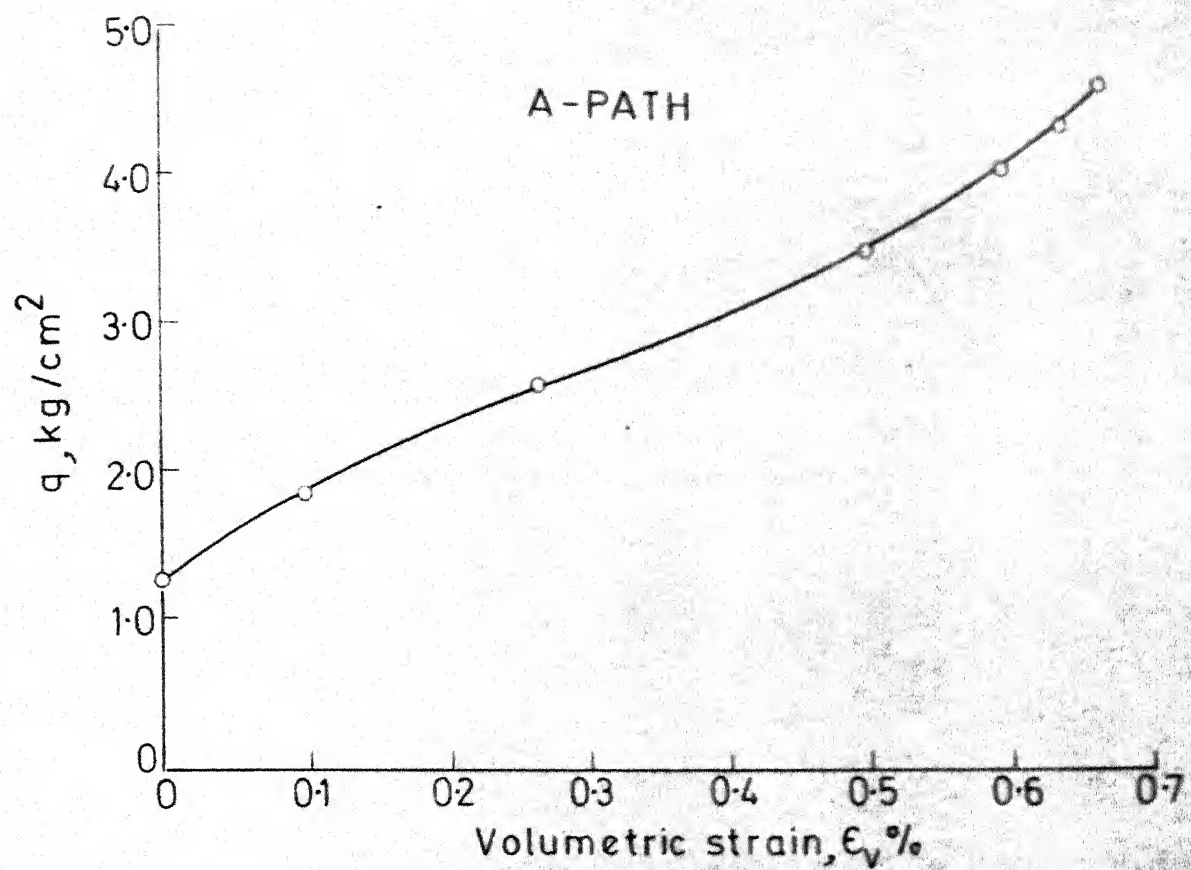


FIG.14 STRESS STRAIN BEHAVIOUR

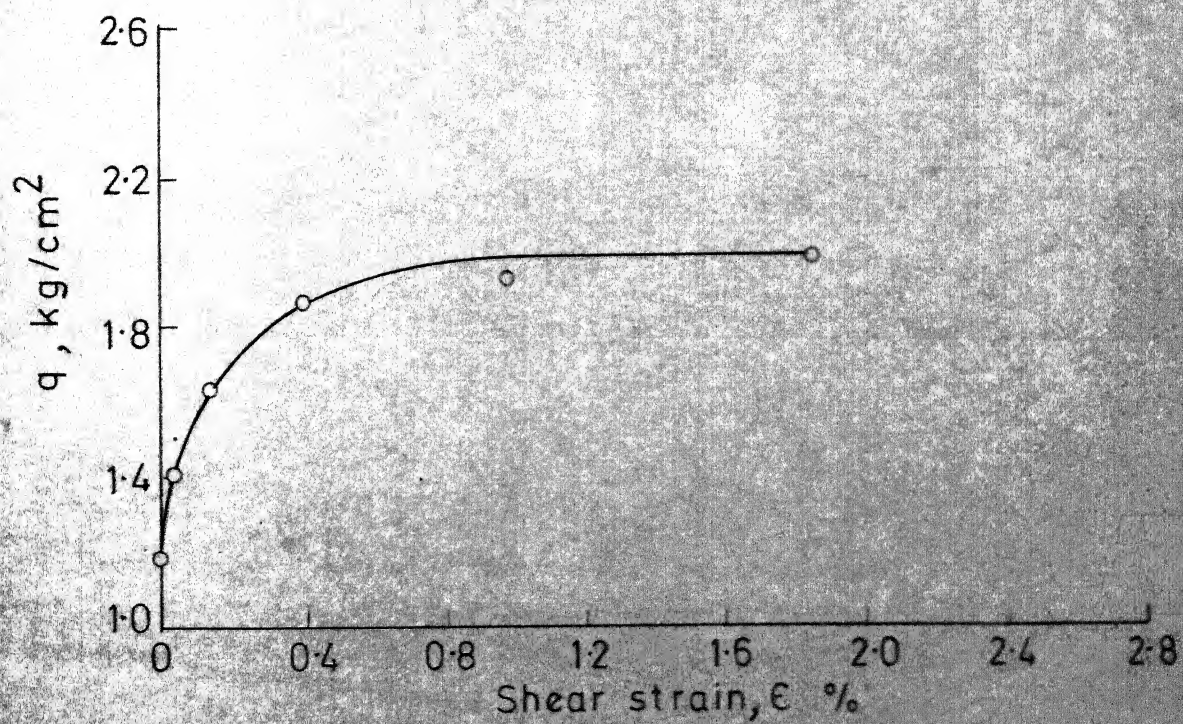
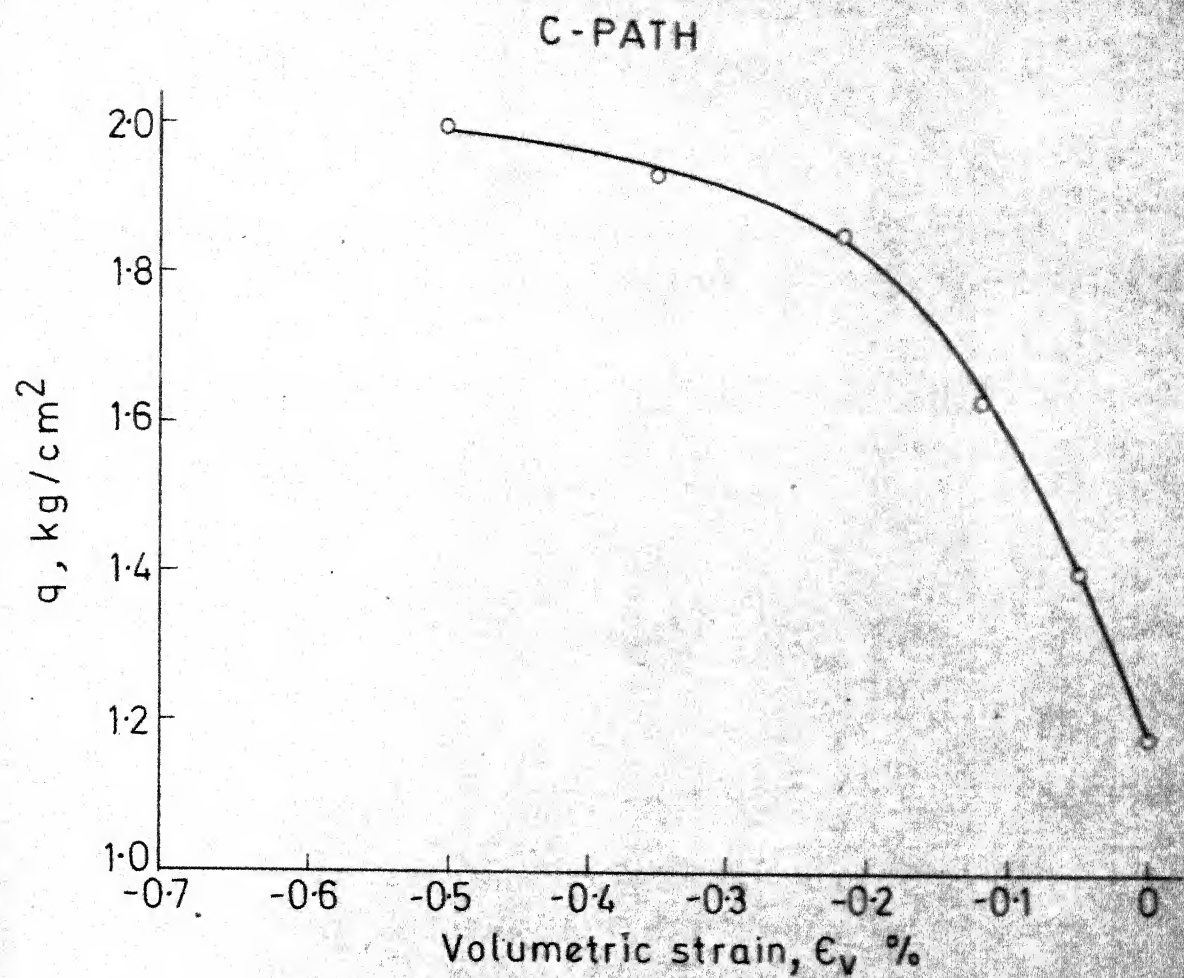


FIG.15 STRESS STRAIN BEHAVIOUR

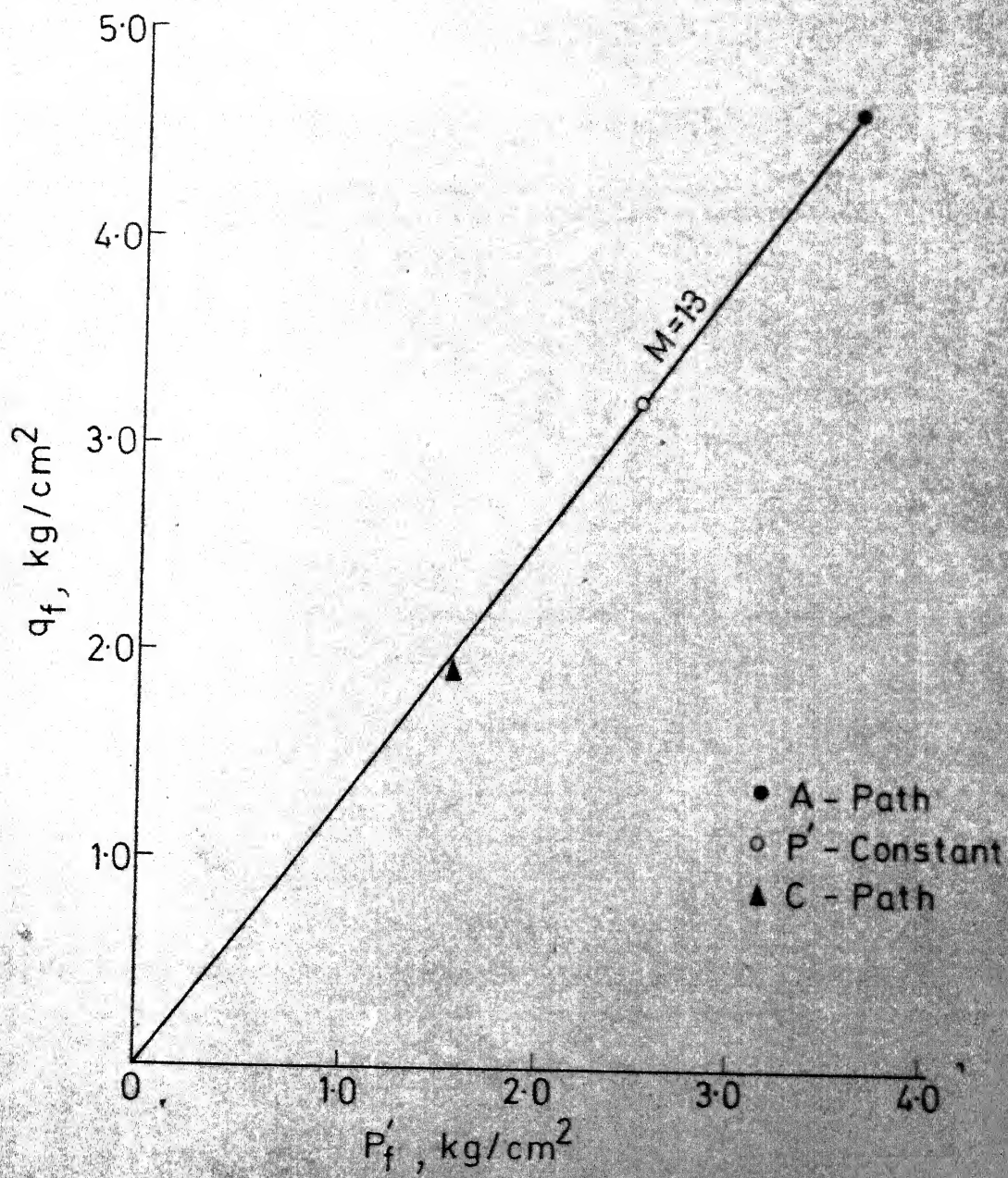
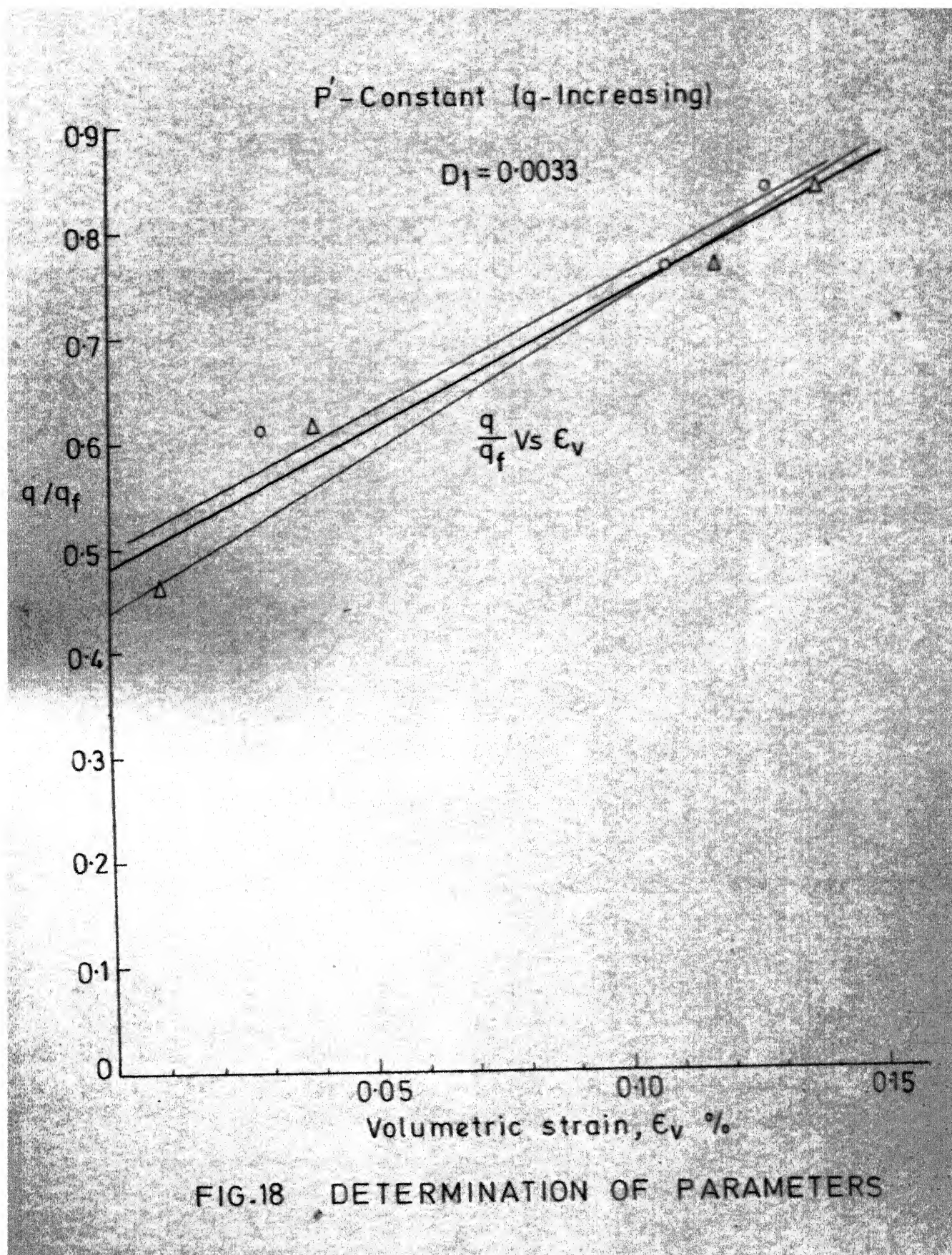


FIG.17 FAILURE ENVELOPE



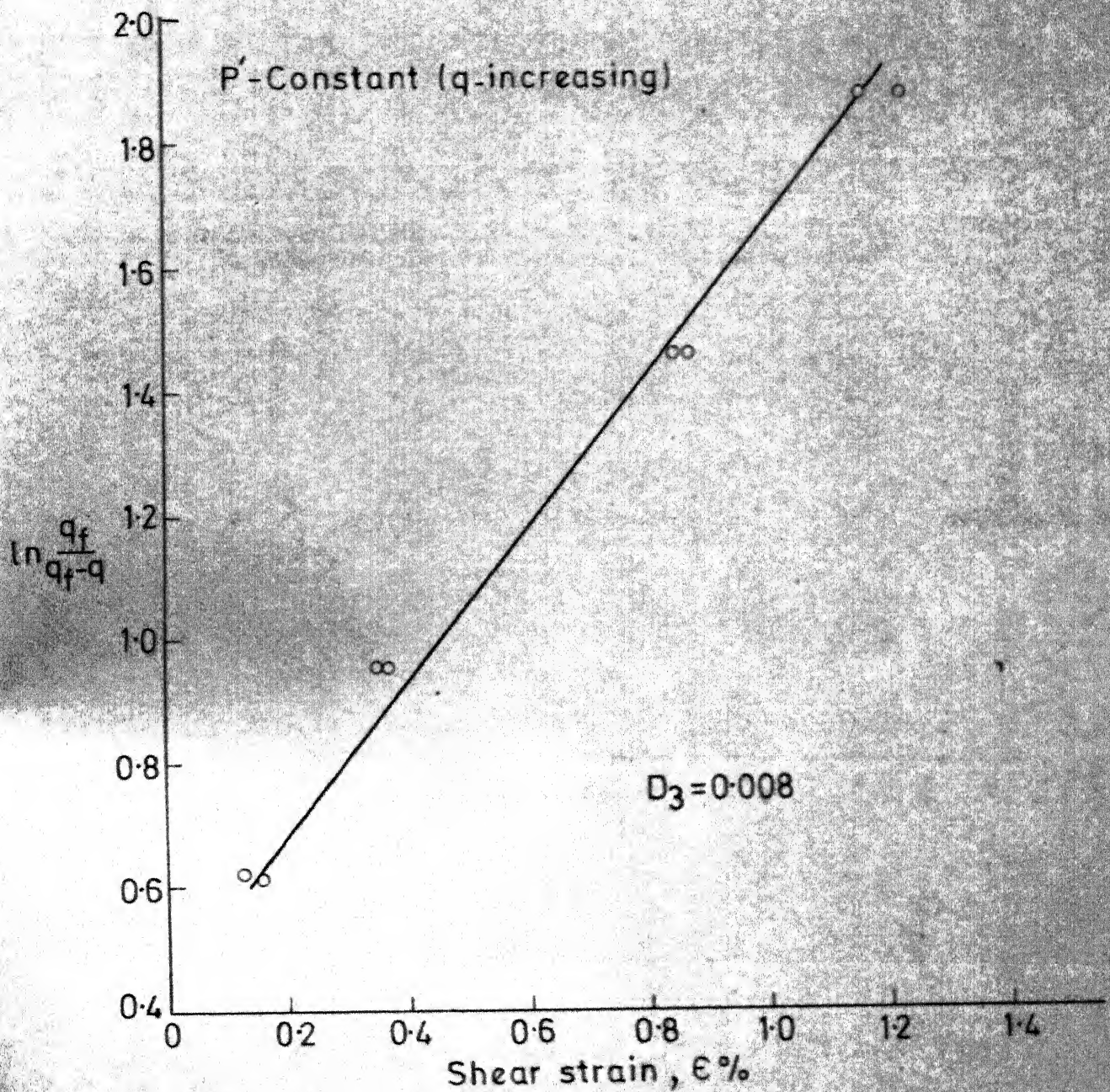
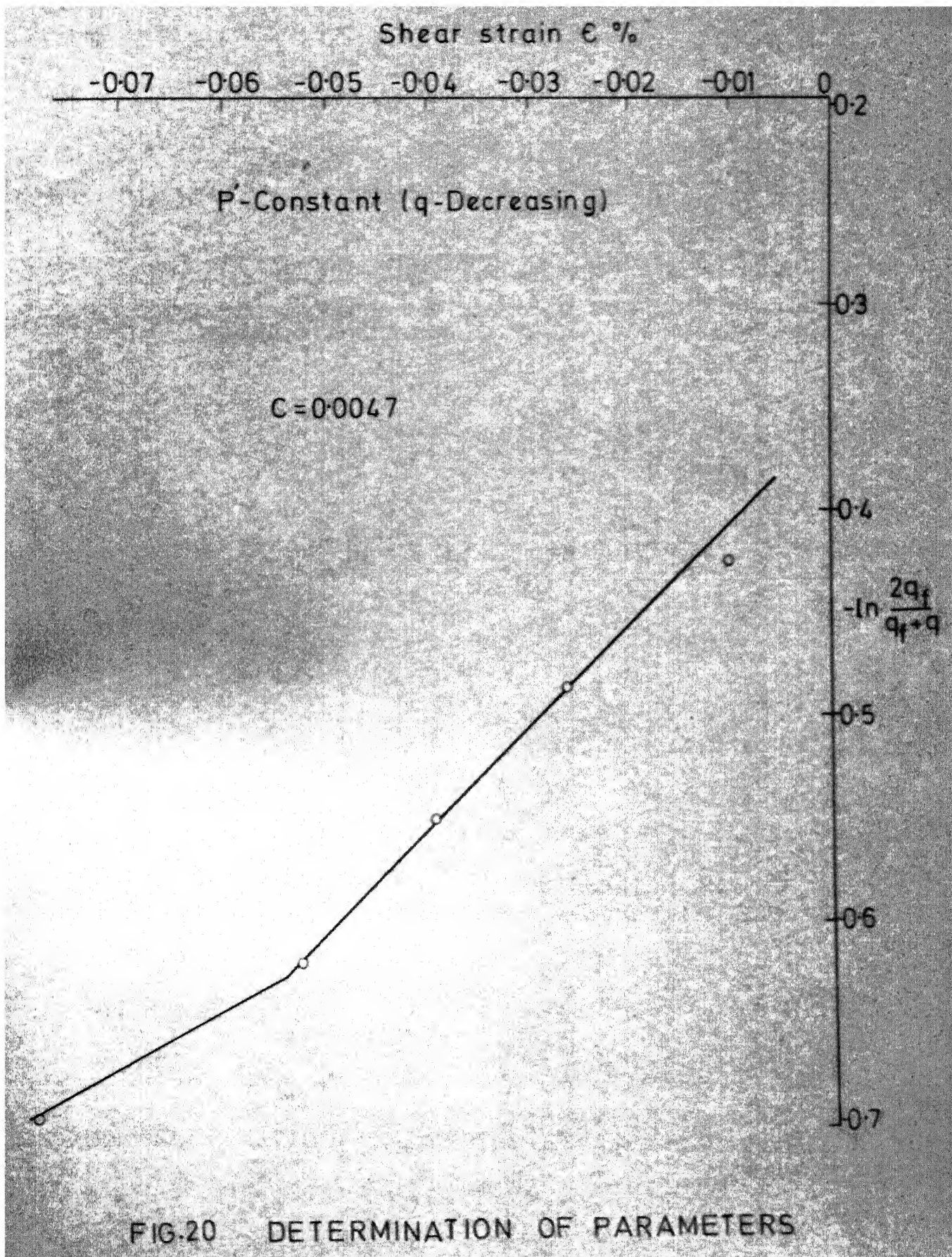


FIG.19 DETERMINATION OF PARAMETERS



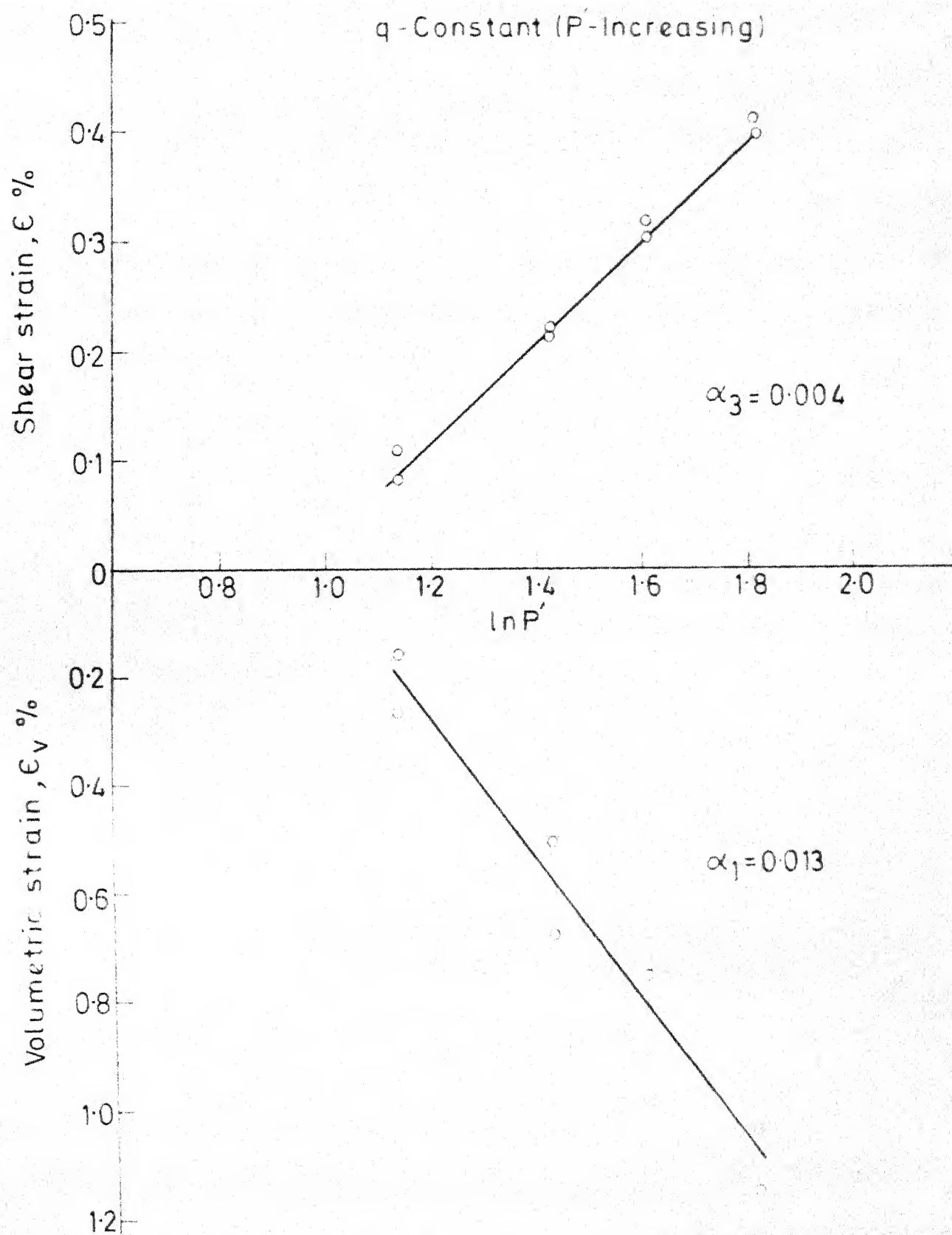


FIG. 21 DETERMINATION OF PARAMETERS

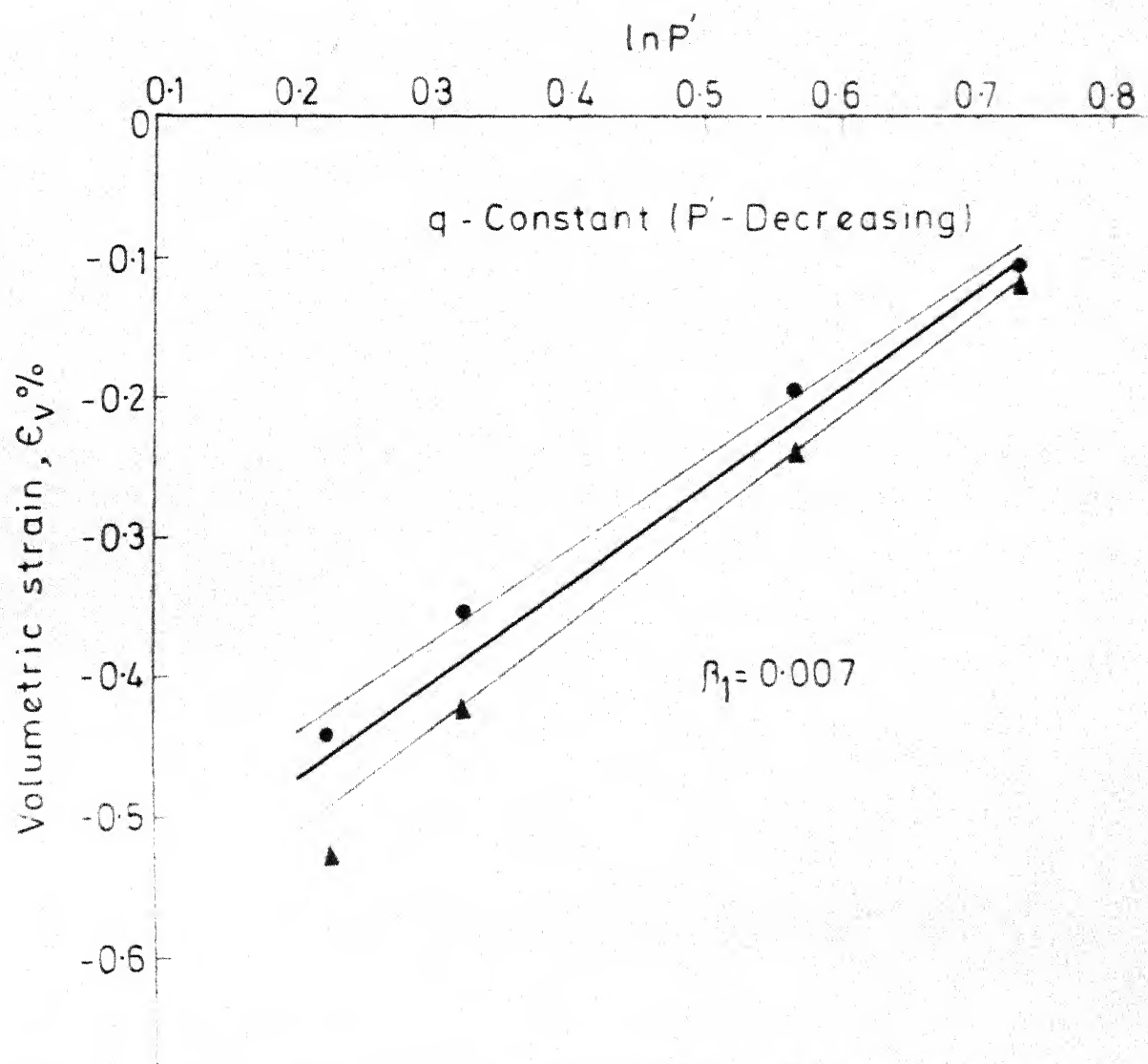


FIG.22 DETERMINATION OF PARAMETERS

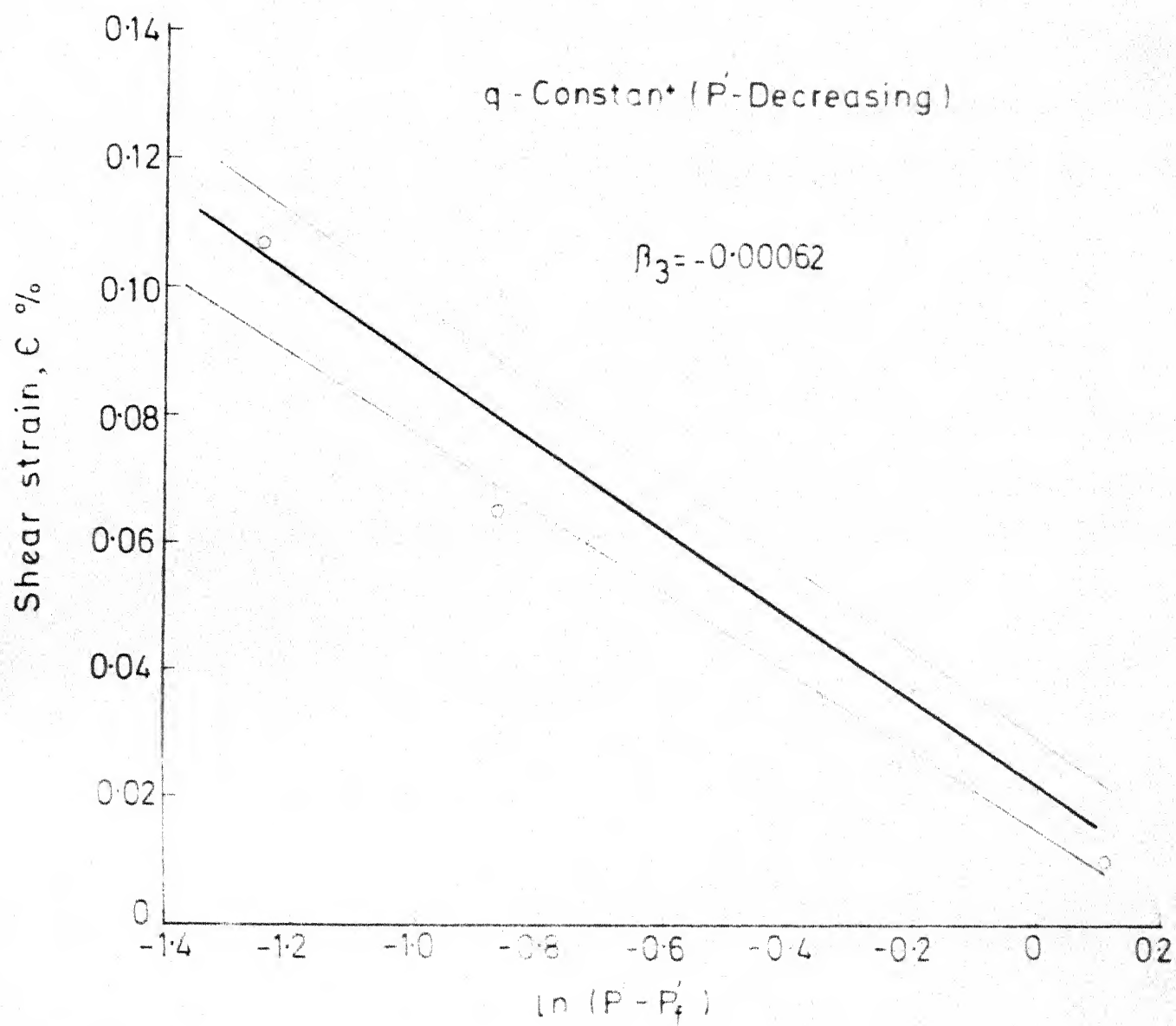


FIG.23 DETERMINATION OF PARAMETER

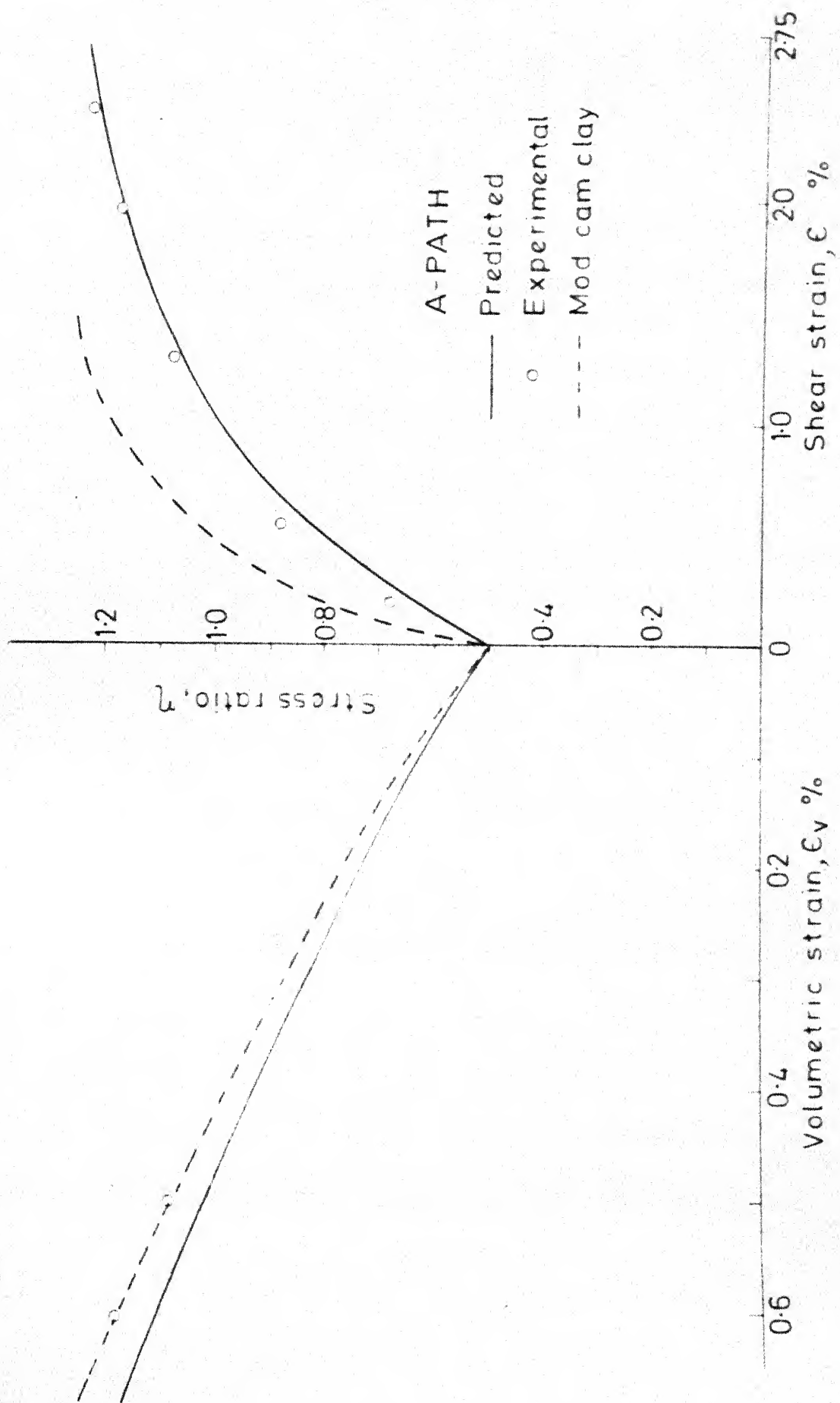


FIG.24 COMPARISON OF STRAINS PREDICTED BY VARIOUS MODELS

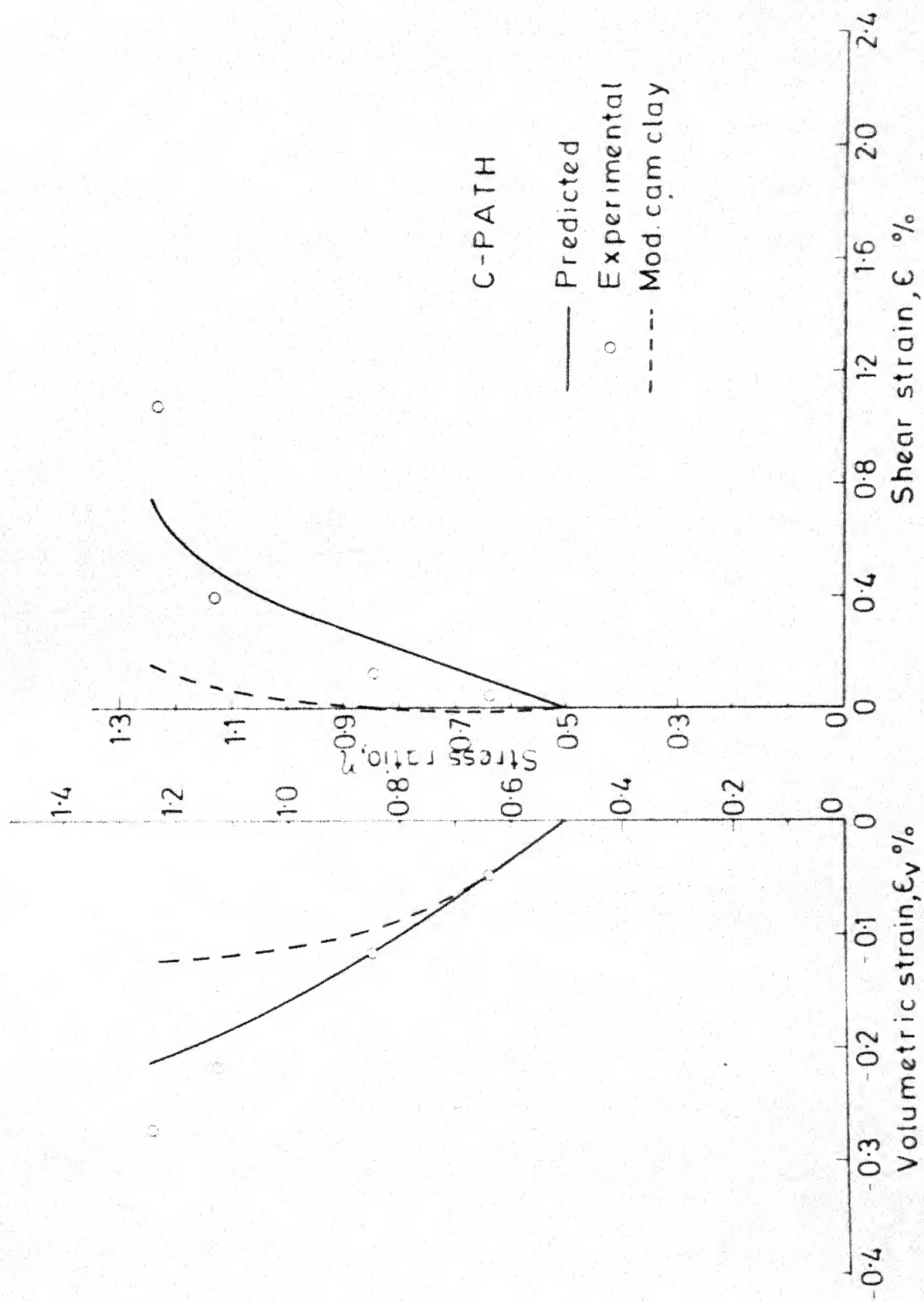


FIG.25 COMPARISON OF STRAINS PREDICTED BY VARIOUS MODELS

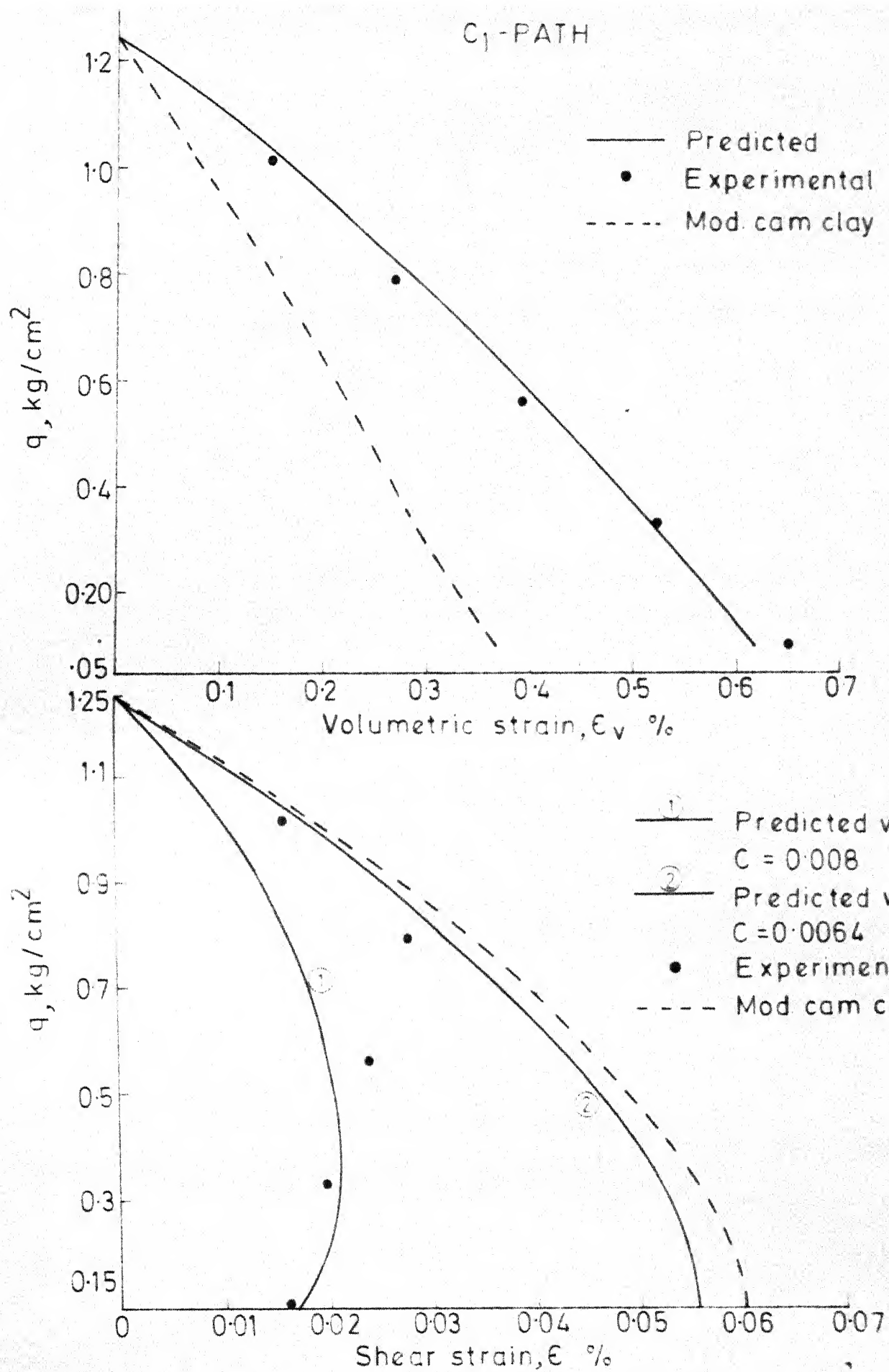


FIG.26 STRESS-STRAIN BEHAVIOUR

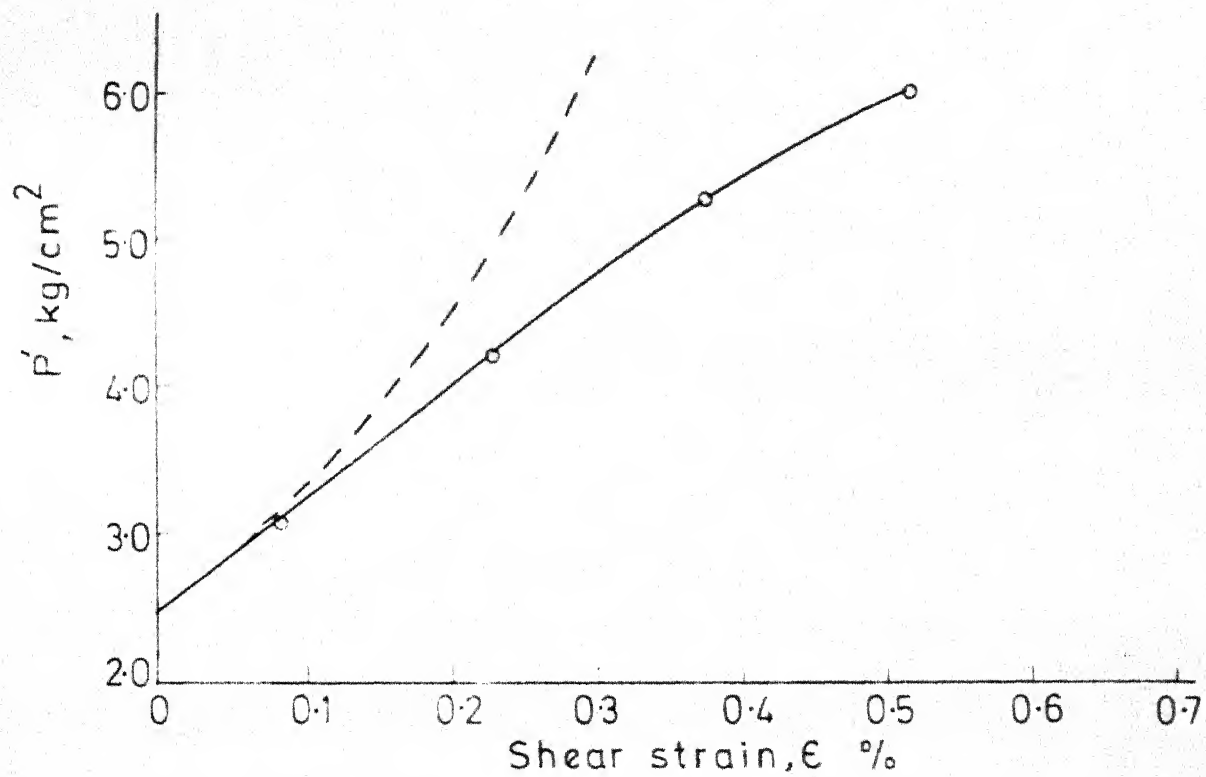
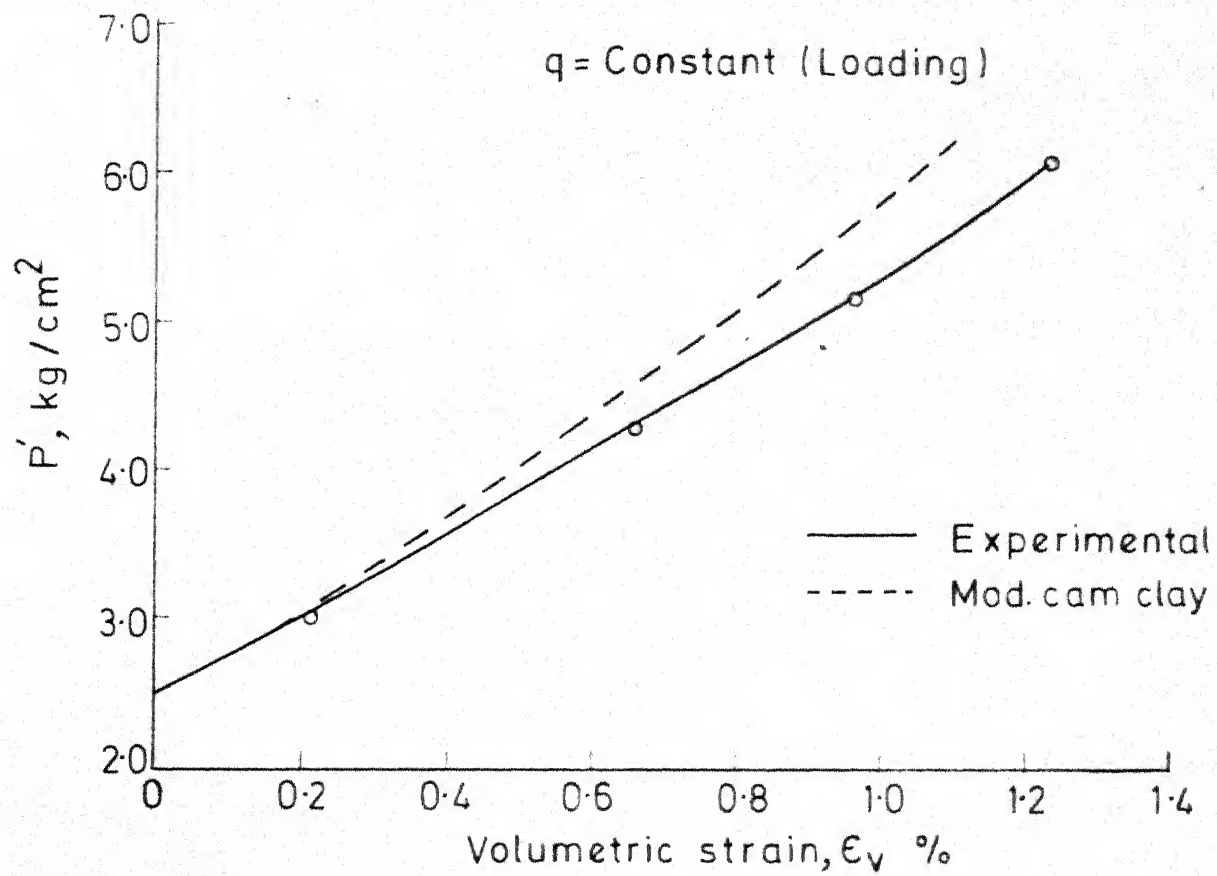


FIG.27 STRESS STRAIN BEHAVIOUR

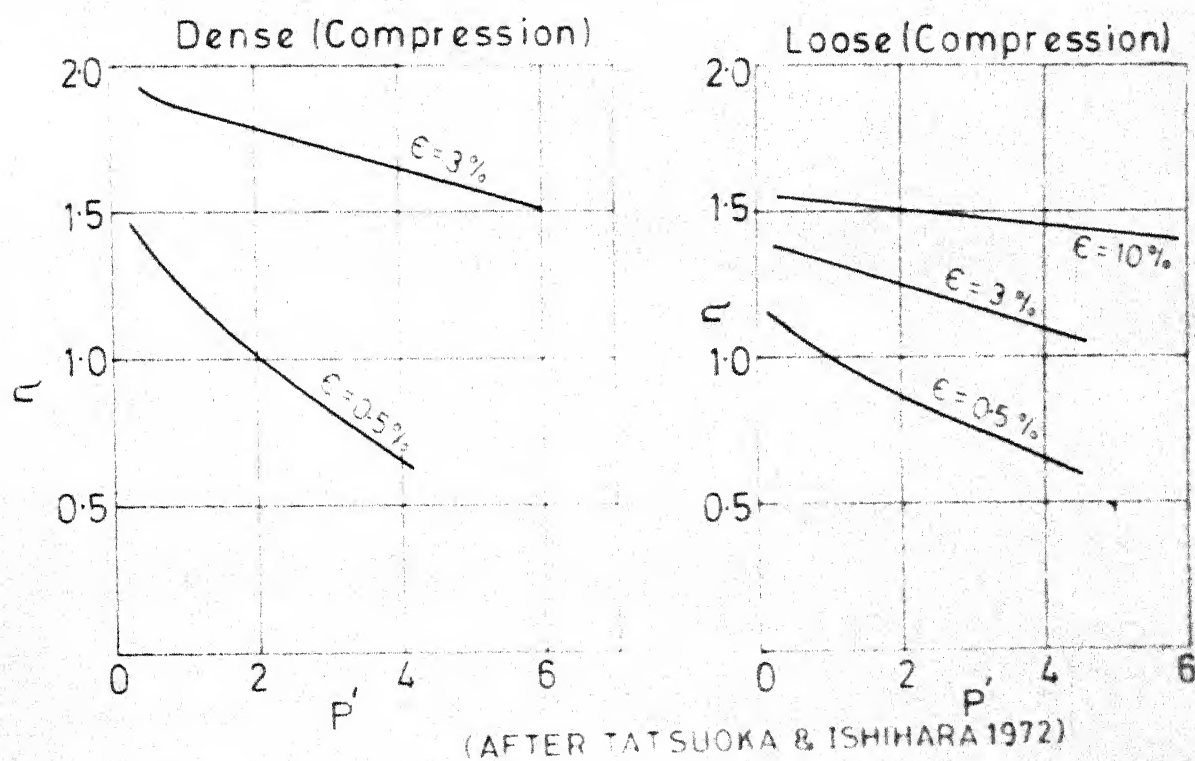
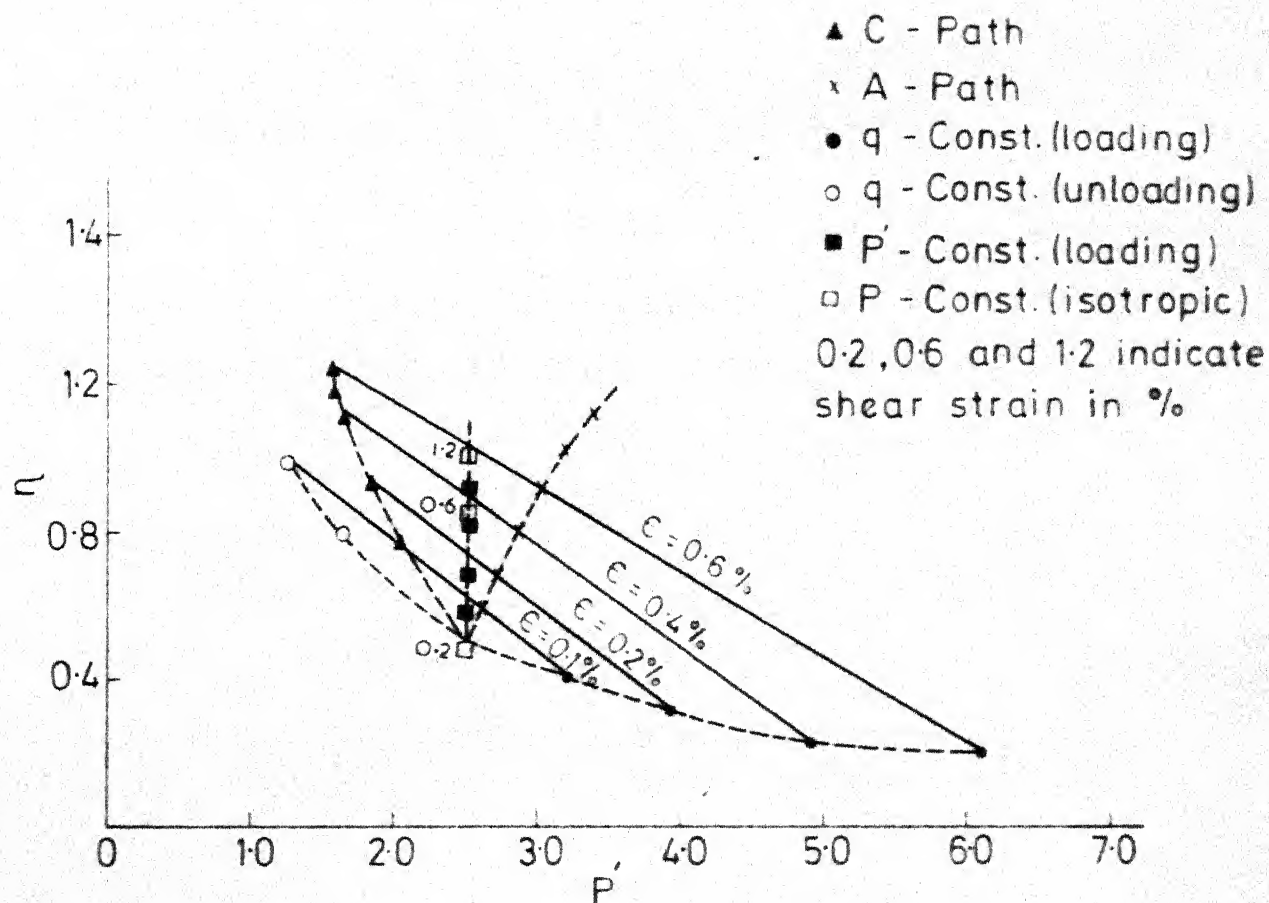


FIG. 28 RELATIONSHIP BETWEEN STRESS RATIO AND SHEAR STRAIN

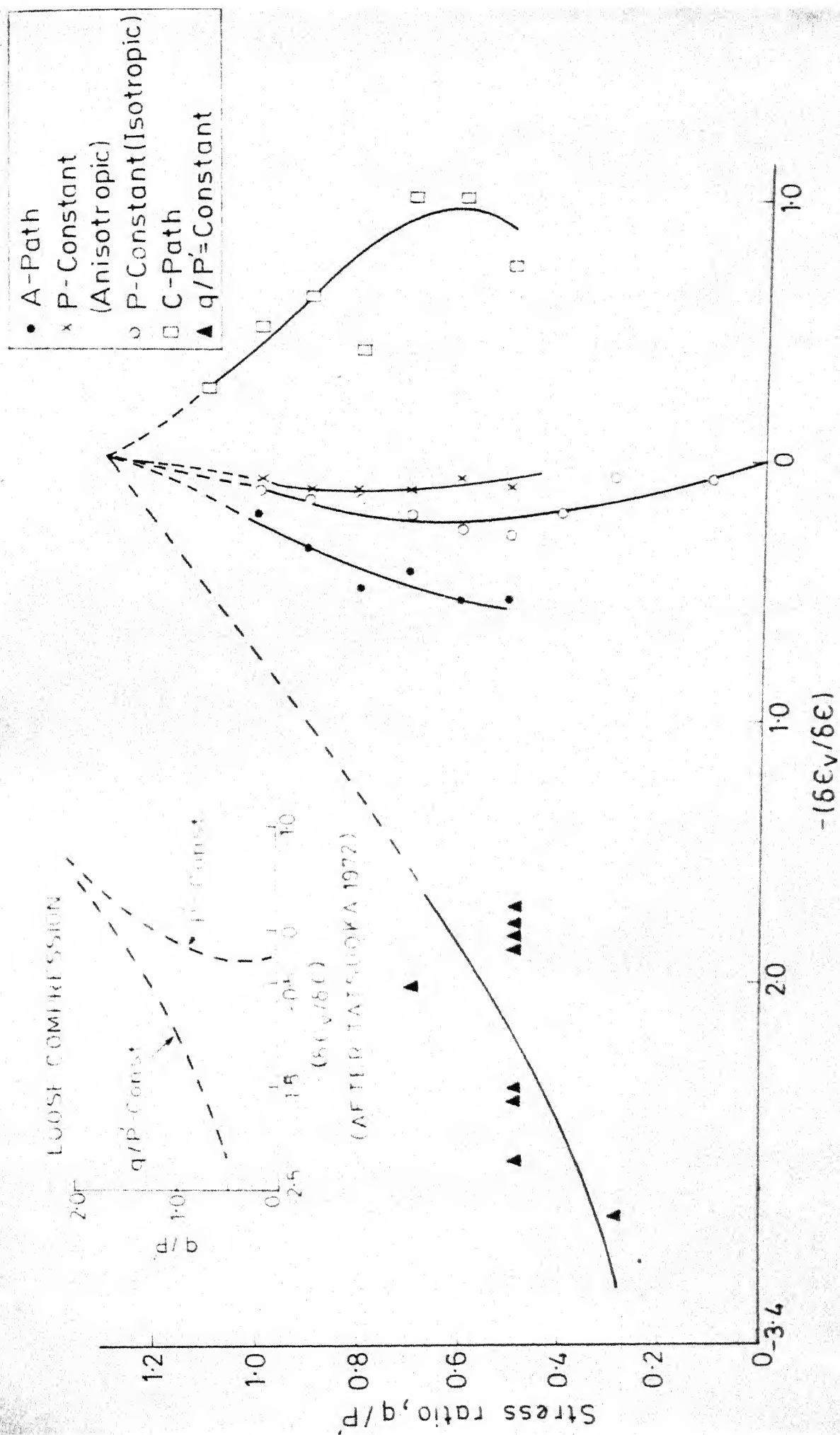


FIG.29 RELATIONSHIP BETWEEN STRESS RATIO q/P' AND STRAIN INCREMENT RATIO

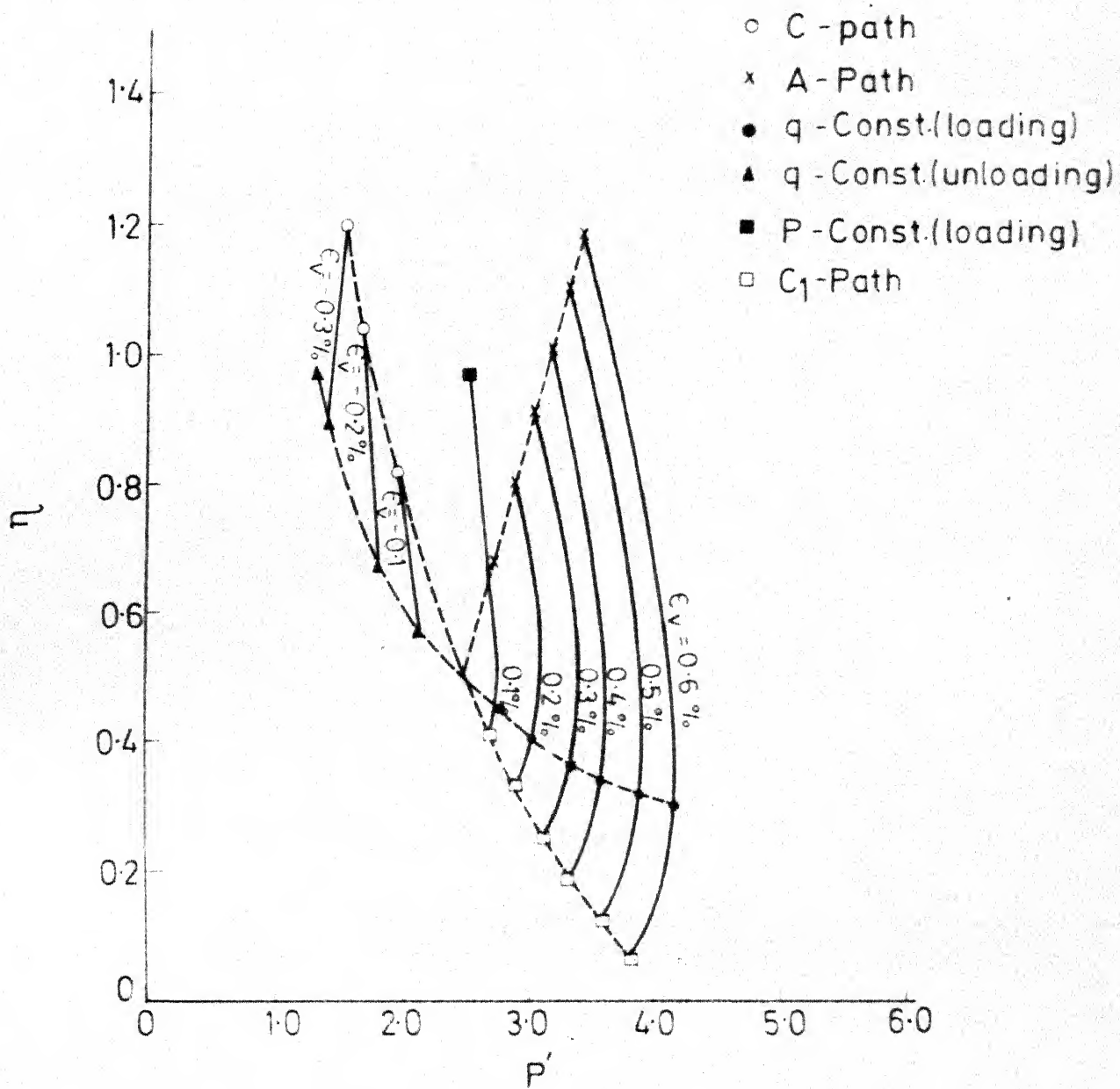


FIG.30 VOLUMETRIC STRAIN CONTOURS IN P' - η SPACE

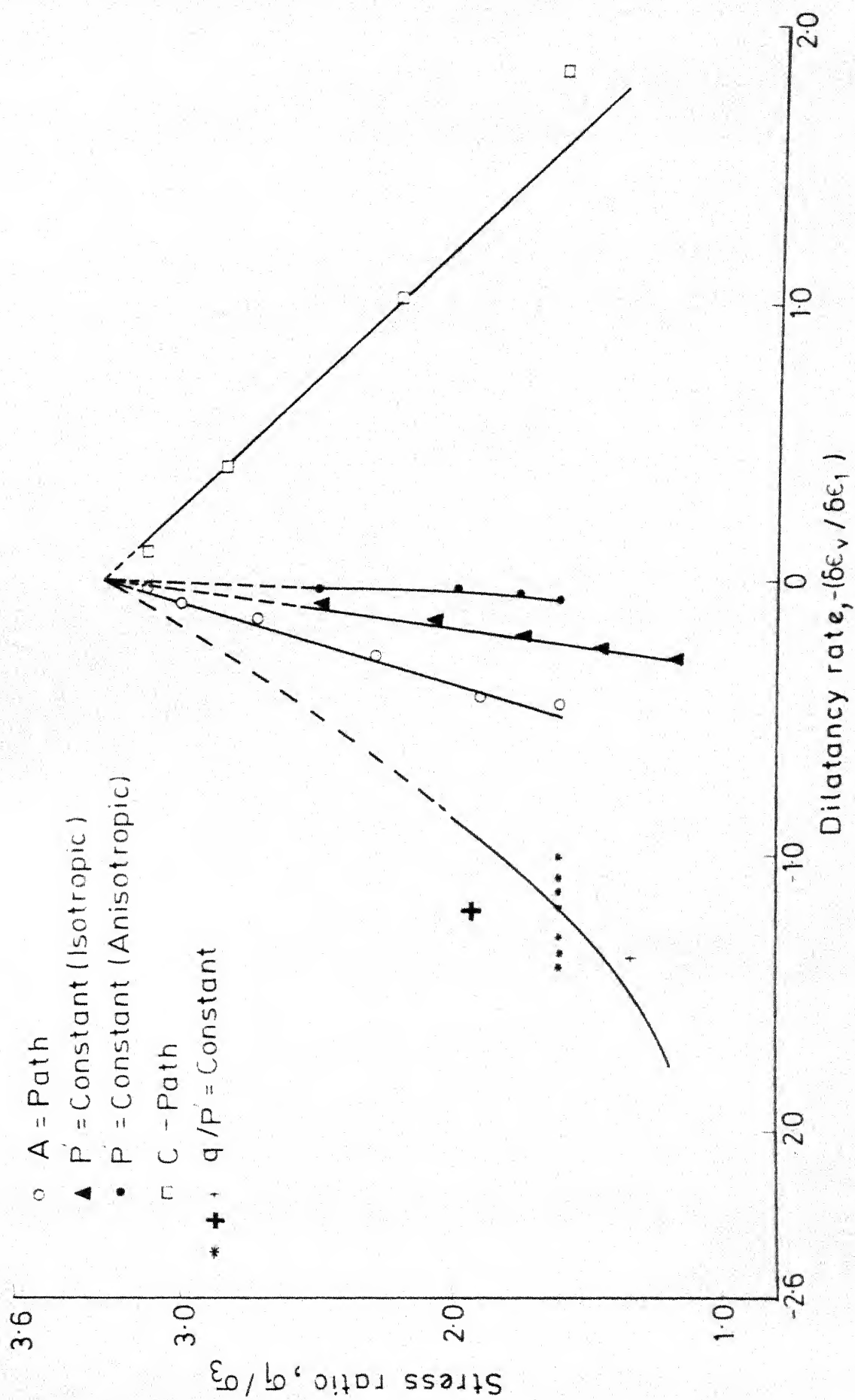


FIG-31 STRESS RATIO STRAIN INCREMENT RATIO RELATION

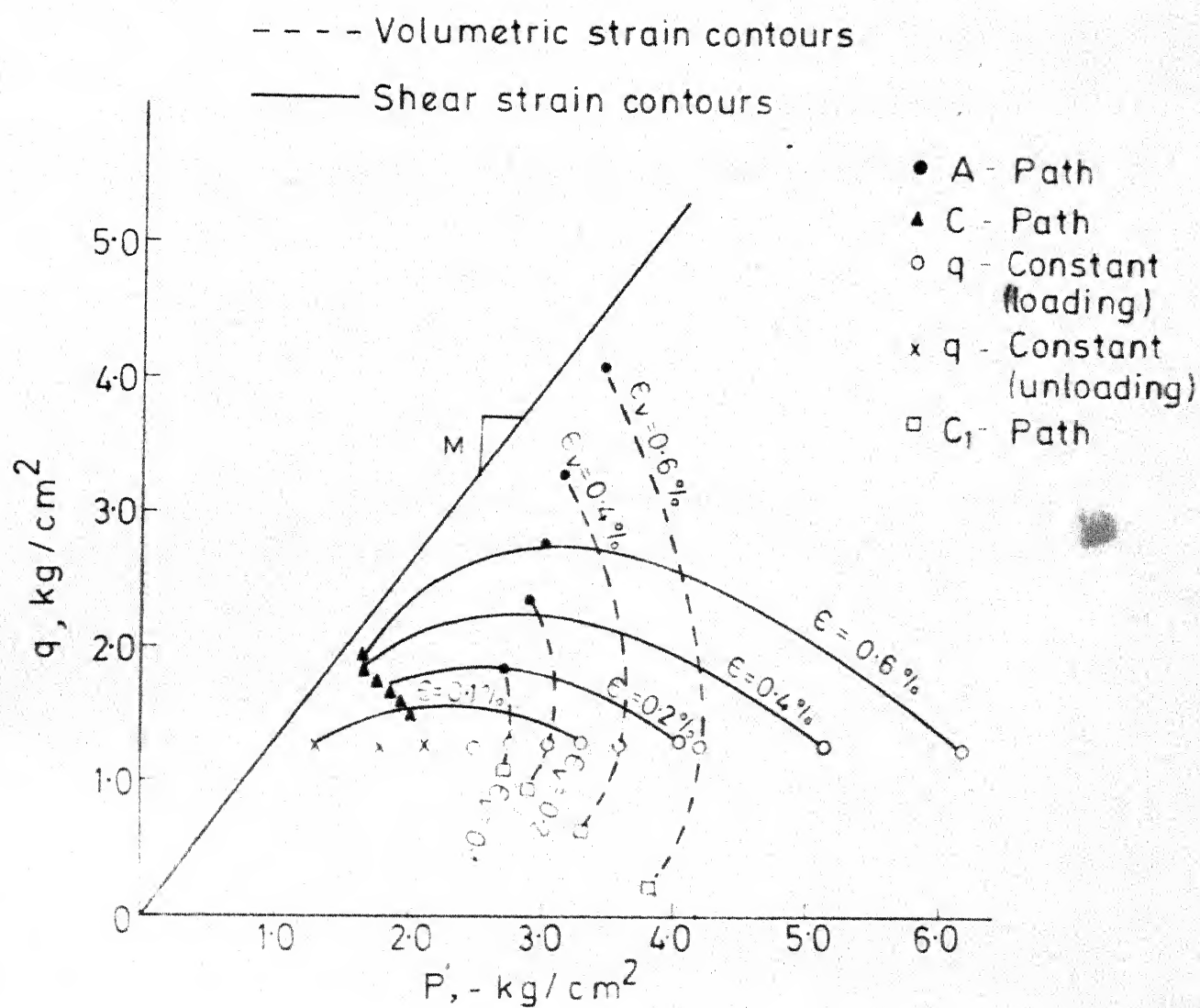


FIG.32 VOLUMETRIC AND SHEAR STRAIN CONTOURS IN P' - q SPACE

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